Logic Programming
Operation Definitions

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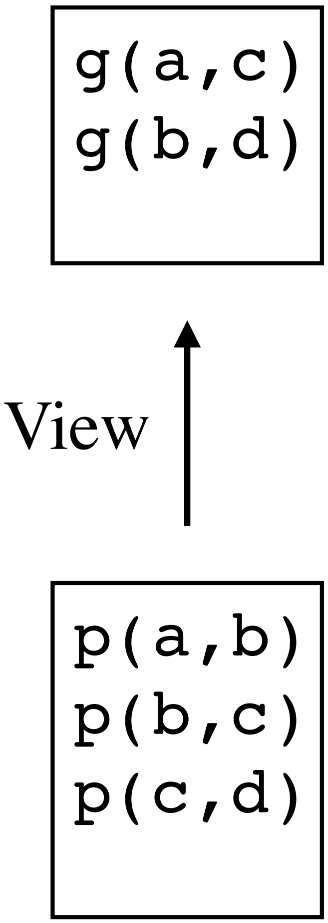
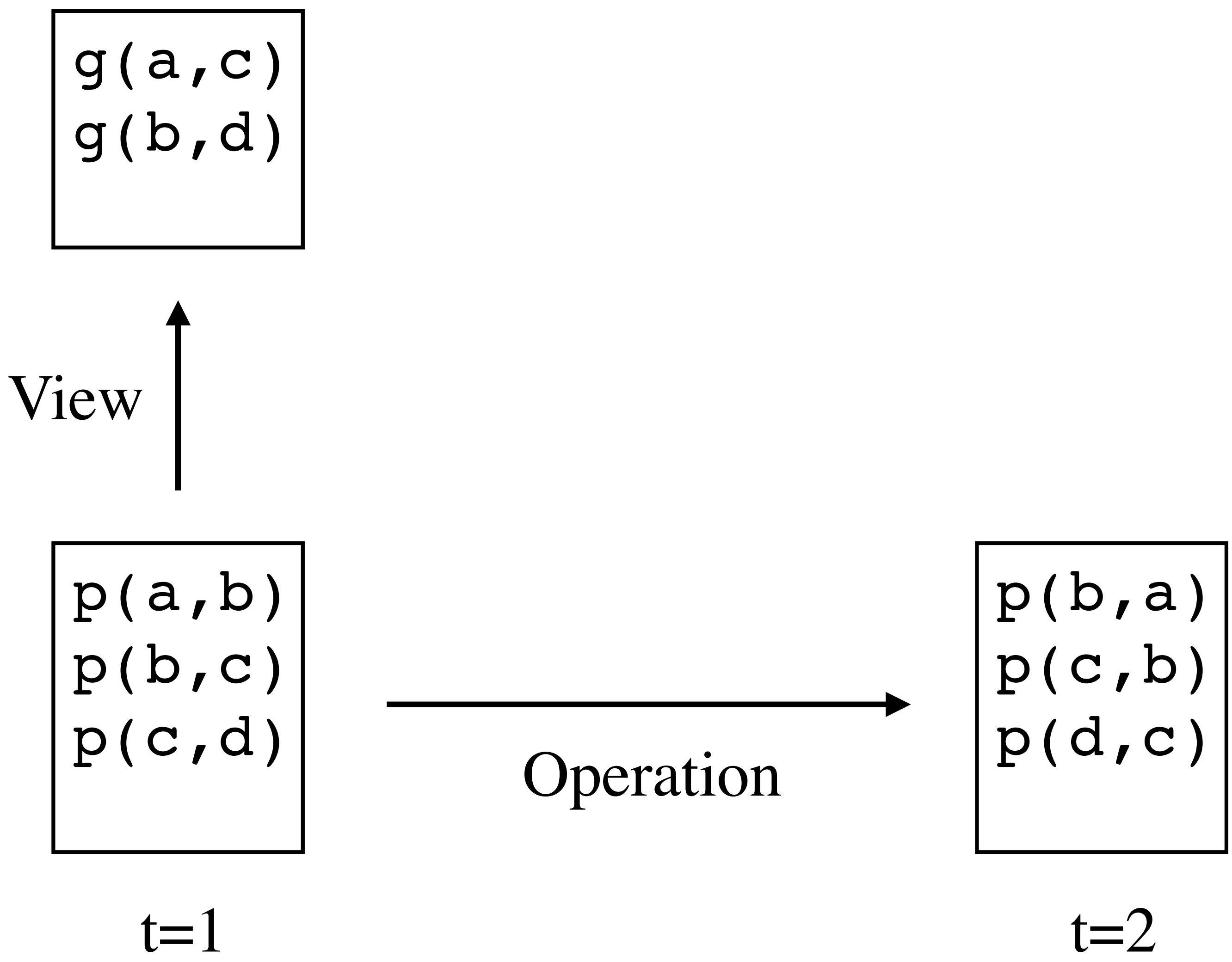
Datasets

```
p(a,b)
p(b,c)
p(c,d)
```

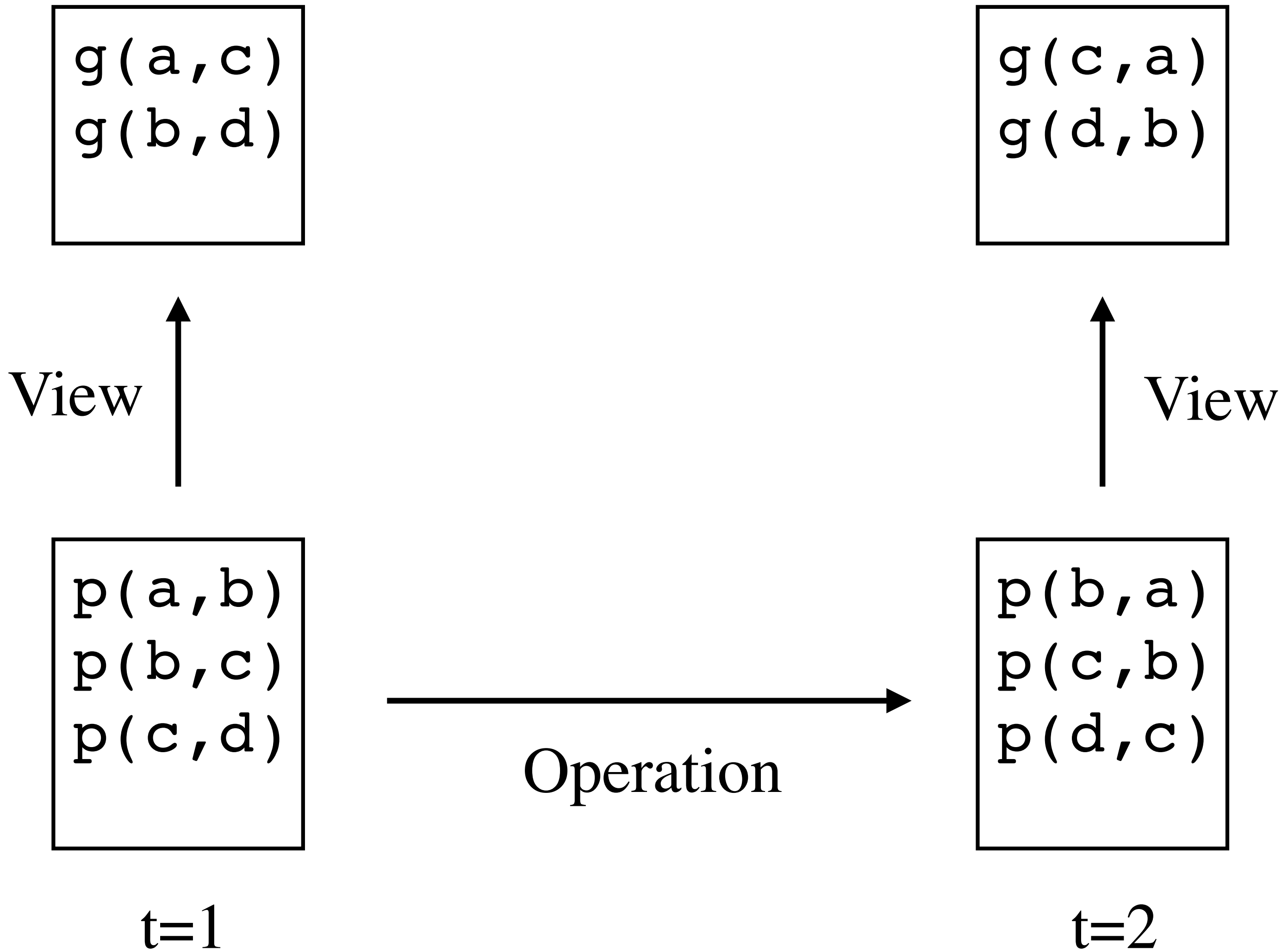
Views



Operations



Operations



g(a,c)
g(b,d)
g(c,a)
g(d,b)
View
View
p(a,b)
p(b,c)
p(c,d)
Operation
p(b,a)
p(c,b)
p(d,c)
t=1
t=2

Operation Definitions

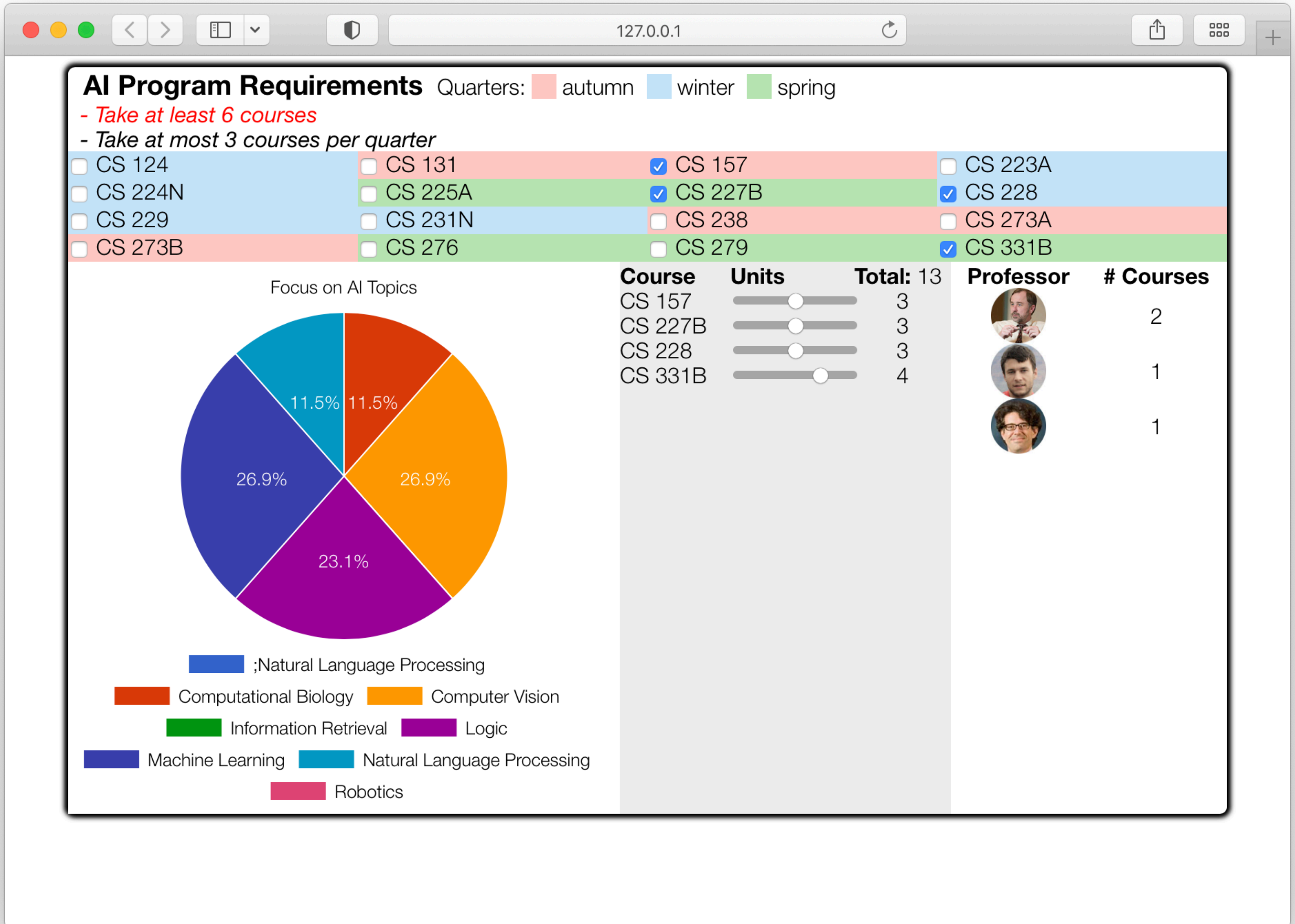
View Definitions

```
r(X,Y) :- p(X,Y) & ~q(Y)
s(X,Y) :- r(X,Y) & r(Y,Z)
```

Operation Definitions

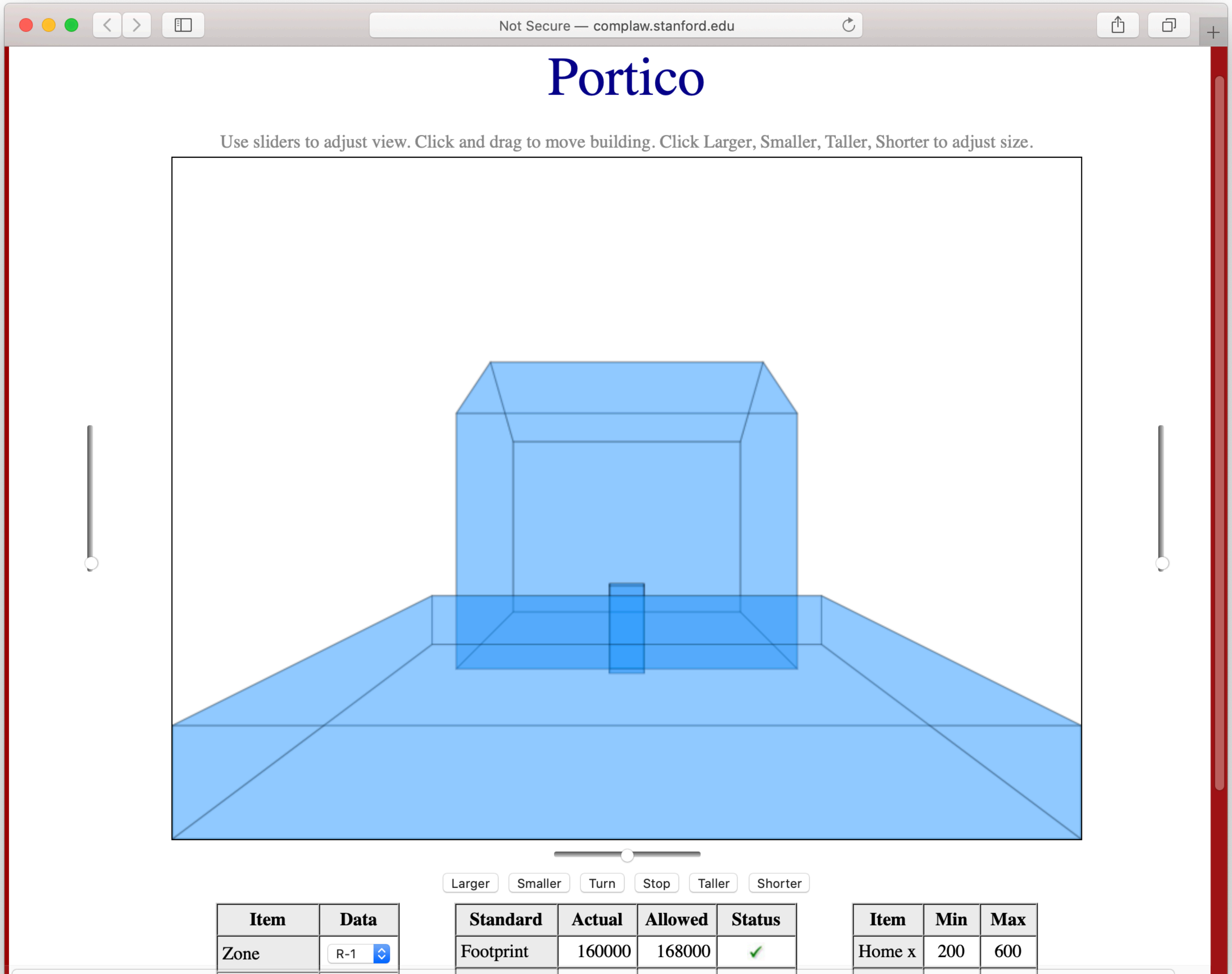
```
flip(X) :: p(X) & ~q(X) ==> ~p(X) & q(X)
flop(X) :: r(X,Y) ==> flip(X) & flop(Y)
```

Program Sheets



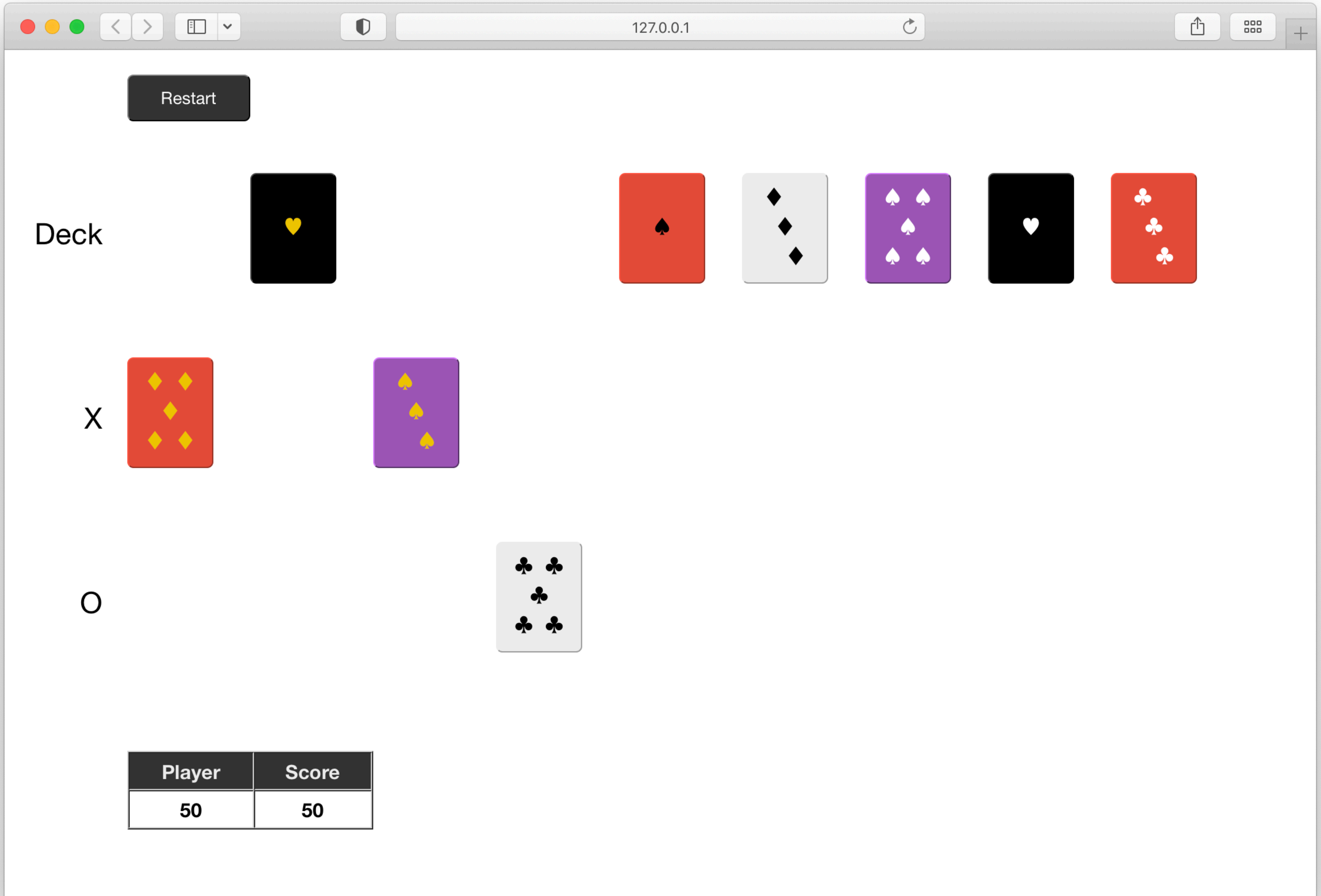
Demonstration

Portico (Symbium)



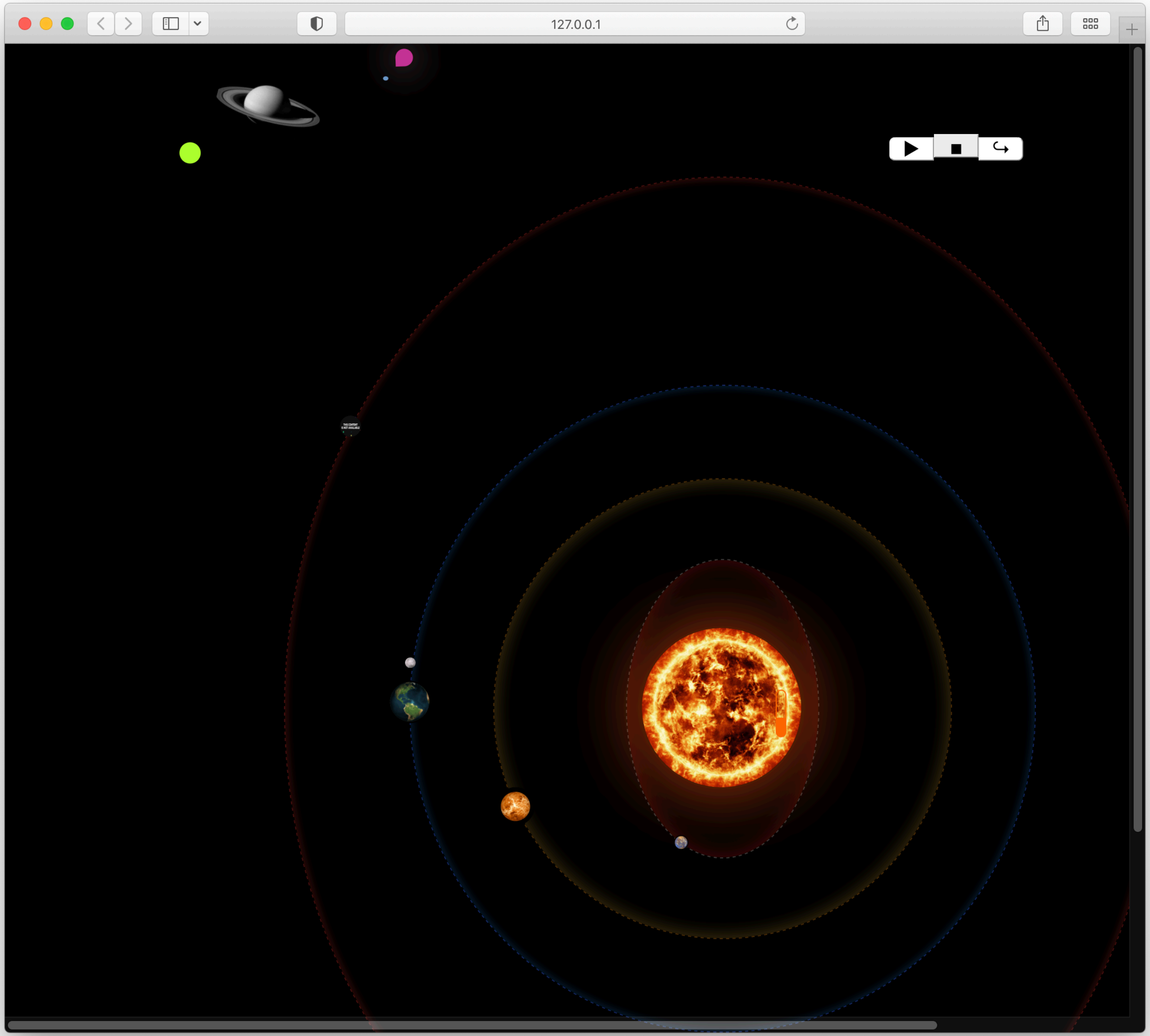
Demonstration

Trifecta



Demonstration

Solar System



Demonstration

Syntax

Operation Constants

Operation constants represent operations.

`tick` - tick of the clock

`click` - click a button on a web page

`stack` - place one block on another

`mark` - place a specific mark in a row and a column

Same spelling conventions as other constants.
Like constructors, and predicates, each has a specific arity.

```
tick/0
click/1
stack/2
mark/3
```

Actions

An **action** is an application of an operation to objects.

In what follows, we denote actions using a syntax similar to that of compound terms, viz. an *n*-ary operation constant followed by *n* terms enclosed in parentheses (as appropriate) and separated by commas.

Examples:

```
tick
click(a)
stack(a,b)
mark(x,2,3)
```

Syntactically, actions are treated as terms.

Operation Definition

```
c(a) :: p(a,b) & q(a) ==> ~q(a) & c(b)
```

head	*conditions*	*effects*
(action)	*(ordinary literals)*	*(base literals or actions)*

Variables

```
c(X) :: p(X,Y) & q(X) ==> ~q(X) & c(Y)
```

Safety

A operation rule is **safe** if and only if every variable in every literal on the right hand side appears in the head or in a positive literal on the left hand side. Also, every variable in a negative literal on the left hand side appears in a prior positive literal.

Safe Operation Rule

```
c(X) ::
  p(X,Y) & ~q(X) ==>
  ~p(X,Y) & q(X) & c(Y)
```

Unsafe Operation Rule

```
c(X) ::
  p(X,Y) & ~q(Z) ==>
  ~p(X,Y) & q(W) & c(Y)
```

Degenerate Rules

Degenerate Rule

```
c(X) :: true ==> ~p(X) & q(X)
```

Shorthand

```
c(X) :: ~p(X) & q(X)
```

Dynamic Logic Programs

An *operation definition* is a finite collection of operation rules with the same operation in the head.

Example

```
c(X) :: p(X) & q(X)
c(X) :: ~r(X) ==> ~p(X) & r(X)
```

A *dynamic logic program* is a collection of view definitions and operation definitions.

Semantics

Intuition

Given a dynamic logic program, the result of applying an action to a dataset is the dataset that results from

(1) *deleting all of the negative effects* of the action

and then

(2) *adding in all of the positive effects*.

Active and Inactive Rule Instances

Given a ruleset Ω with dataset Δ *and* a set Γ of actions, an *instance* of an operation rule in Ω is **active** if and only if

(1) the head of the rule is in Γ

(2) the conditions of the rule are all true in Δ.

Otherwise, the instance is **inactive**.

Example

Data: `p(a),p(b),p(c),q(a),q(b),q(c),r(b)`

Rule:

```
u(X) :: p(X) & q(X) & ~r(X) ==> ~p(X) & r(X)
```

Action: `u(a)`

Active Instance:

```
u(a) :: p(a) & q(a) & ~r(a) ==> ~p(a) & r(a)
```

Inactive Instances:

```
u(b) :: p(b) & q(b) & ~r(b) ==> ~p(b) & r(b)
u(c) :: p(c) & q(c) & ~r(c) ==> ~p(c) & r(c)
```

Expansion

The **expansion*** of an action set with respect to a rule set is the set of all effects in any active instance of any operation definition.

The **positive updates** of an action with respect to a rule set are the positive literals in the expansion.

The **negative updates** of an action with respect to a rule set are the negative literals in the expansion.

**Simple version*

Example

Data: `p(a),p(b),p(c),  q(a),q(b),q(c),    r(b)`

Rule:

```
u(X) :: p(X) & q(X) & ~r(X) ==> ~p(X) & r(X)
```

Action: `u(a)`

Active Instance:

```
u(a) :: p(a) & q(a) & ~r(a) ==> ~p(a) & r(a)
```

Expansion: `~p(a), r(a)`
Negative Update: `p(a)`
Positive Update: `r(a)`

Result

Given a rule set, the **result** of applying an action set to dataset Δ is the set consisting of all factoids in Δ *minus* the negative updates *plus* the positive updates.

$$\Delta - \mathit{negatives} \cup \mathit{positives}$$

Example

Data: `p(a),p(b),p(c), q(a),q(b),q(c), r(b)`

Rule:

`u(X) :: p(X) & q(X) & ~r(X) ==> ~p(X) & r(X)`

Action: `u(a)`

Negative Updates: `p(a)`
Positive Updates: `r(a)`

Result: `p(b),p(c), q(a),q(b),q(c), r(a),r(b)`

Multiple Rules

Dataset: `p(a),p(b),p(c), q(a),q(b),q(c), r(b)`

Rule:

```
u(X) :: p(X) & q(X) & ~r(X) ==> ~p(X)
u(X) :: p(X) & q(X) & ~r(X) ==> r(X)
```

Action: `u(a)`

Negative effects: `p(a)`

Positive effects: `r(a)`

Result: `p(b),p(c), q(a),q(b),q(c), r(a),r(b)`

Weird Case

Dataset: `{p(a),p(b),p(c), q(a),q(b),q(c)}`

Rule:

```
u(X) :: p(X) & q(X) ==> ~r(X)
u(X) :: p(X) & q(X) ==> r(X)
```

Action: `u(a)`

Negative effects: `r(a)`

Positive effects: `r(a)`

Result: `p(a),p(b),p(c), q(a),q(b),q(c), r(a)`

Simultaneous Actions

Data: `p(a),p(b),p(c),  q(a),q(b),q(c),    r(b)`

Rule:

```
u(X) :: p(X) & q(X) & ~r(X) ==> ~p(X) & r(X)
```

Actions: `u(a),u(b),u(c)`

Active Instances:

```
u(a) :: p(a) & q(a) & ~r(a) ==> ~p(a) & r(a)
u(c) :: p(c) & q(c) & ~r(c) ==> ~p(c) & r(c)
```

Inactive Instance:

```
u(b) :: p(b) & q(b) & ~r(b) ==> ~p(b) & r(b)
```

Simultaneous Actions

Data: `p(a),p(b),p(c), q(a),q(b),q(c), r(b)`

Rule:

`u(X) :: p(X) & q(X) & ~r(X) ==> ~p(X) & r(X)`

Actions: `u(a),u(b),u(c)`

Expansion: `~p(a), ~p(c), r(a), r(c)`
Negative Updates: `p(a),p(c)`
Positive Updates: `r(a),r(c)`

Result: `p(b), q(a),q(b),q(c), r(a),r(b),r(c)`

Derived Actions

Data: `p(a),p(b),p(c), q(a),q(b),q(c), r(b)`

Rule:

`u(X) :: p(X) & q(X) ==> ~p(X) & r(X) & u(c)`

Input Action: `u(a)` **Derived action**: `u(c)`

Expansion: `~p(a),~p(c),r(a),r(c),u(a),u(c)`
Negative Updates: {`p(a), p(c)`}
Positive Updates: {`r(a), r(c)`}

Result: `p(b), q(a),q(b),q(c), r(a),r(b),r(c)`

Expansion

Given a rule set Ω and a dataset Δ a set Γ of actions, consider the following series.

$\Gamma_0 = \Gamma$
Γ_{n+1} = the set of all effects of Γ in any active rule instance

The **expansion*** of Γ with respect to Ω and Δ is the fixpoint of this series.

The **positive updates** of an action with respect to a rule set are the positive literals in the full expansion.

The **negative updates** of an action with respect to a rule set are the negative literals in the full expansion.

**Exact version*

Interchange

```
function interchange ()
 {x = y;
  y = x}

[x, y]
[3, 4]

interchange()

[x, y]
[4, 4]


function interchange ()
 {var z = x;
  x = y;
  y = z}
```

Interchange

```
interchange ::
  val(x,X) & val(y,Y) ==>
    ~val(x,X) & ~val(y,Y) &
    val(x,Y) & val(y,X)
```

```
val(x,3)
val(y,4)
```

```
Execute: interchange
```

```
val(x,4)
val(y,3)
```

Production Systems

A **production system** is a set of condition-action rules. On each step in the execution of a production system, an active rule is chosen and the actions are performed. The cycle then repeats on the new state.

```
if p(X), then del p(X) and add q(X)
if q(X), then del q(X) and add p(X)
```

Before: {`p(a),q(b)`}
Step 1: {`q(a),q(b)`}
Step 2: {`p(a),q(b)`} *or* {`p(b),q(a)`}

When do we stop?

Dynamic Logic Programs

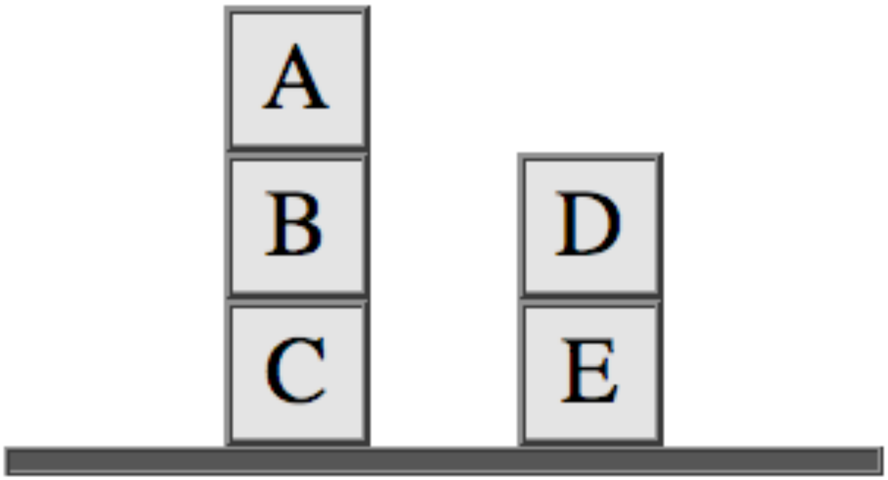
Dynamic logic programs differ from production systems in that all active transition rules "fire" at the same time. (1) All updates are computed *before* any changes are made, and (2) all changes are made simultaneously.

```
tick :: p(X) ==> ~p(X) & q(X)
tick :: q(X) ==> ~q(X) & p(X)
```

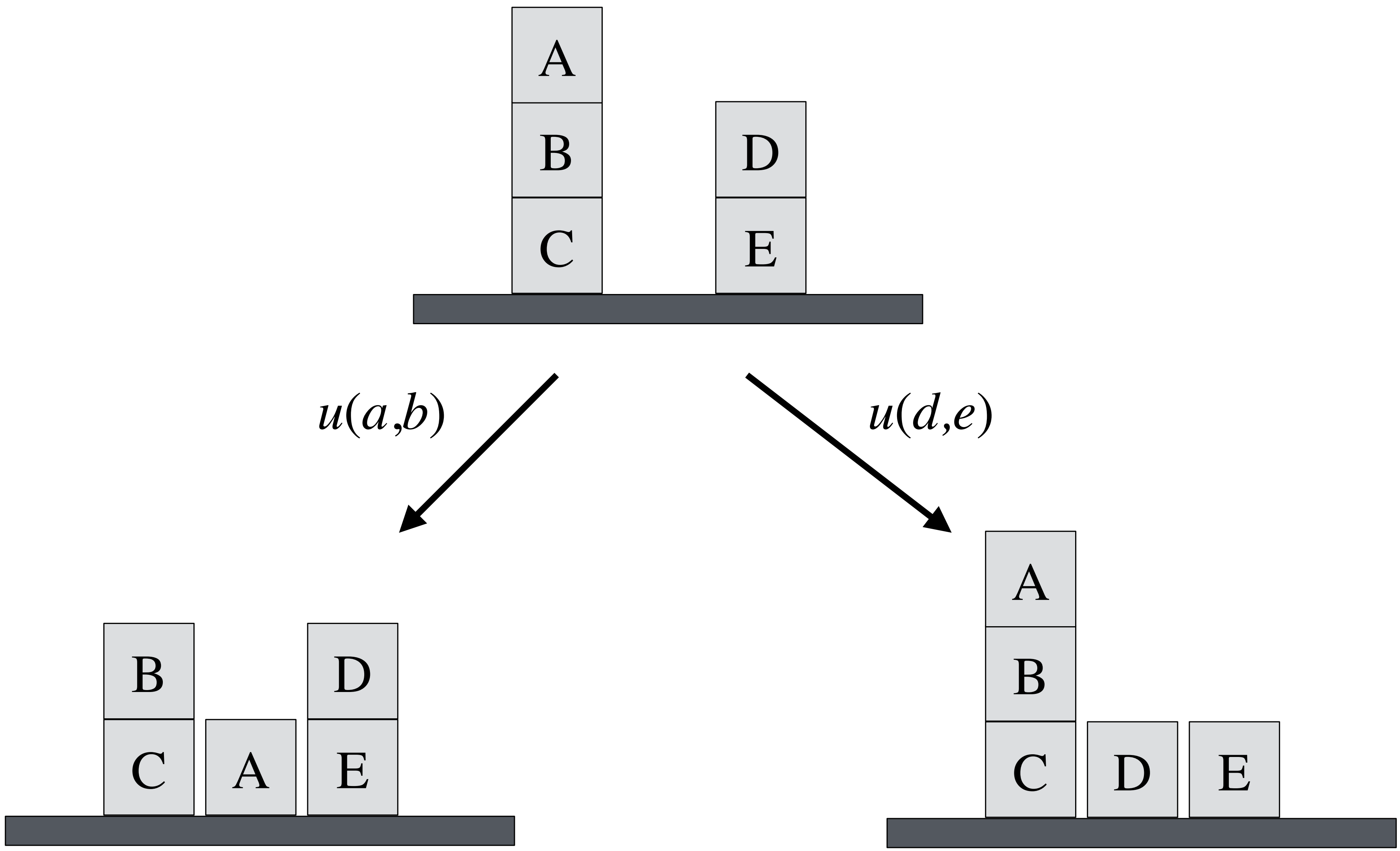
Before: `{p(a), q(b)}`
After: `{p(b), q(a)}`

Blocks World

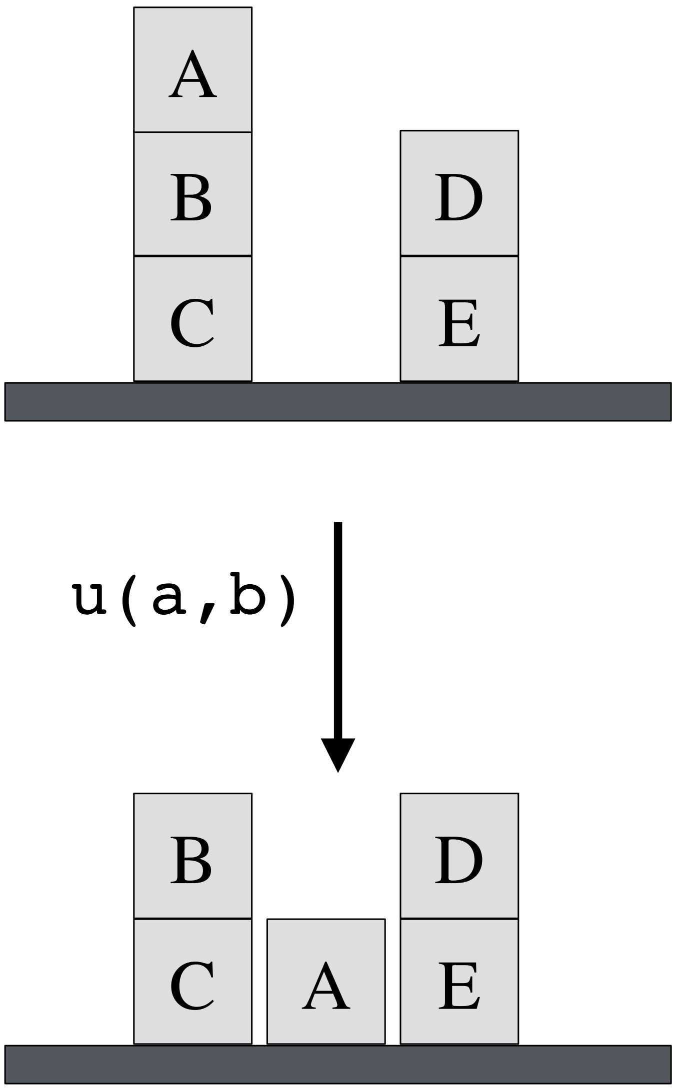
Blocks World



External Actions



Describing States




```
clear(a)
on(a,b)
on(b,c)
on(d,e)
  ...
```

```
clear(a)
table(a)
clear(b)
on(b,c)
on(d,e)
  ...
```

Operation Definitions

Operations:

`u(x,y)` means that x is moved from y to the table.

`s(x,y)` means that x is moved from the table to y.

Operation Definitions:

```
u(X,Y) ::
    clear(X) & on(X,Y)
    ==> ~on(X,Y) & table(X) & clear(Y)
```

Operation Definitions

Operations:

`u(x,y)` means that x is moved from y to the table.

`s(x,y)` means that x is moved from the table to y.

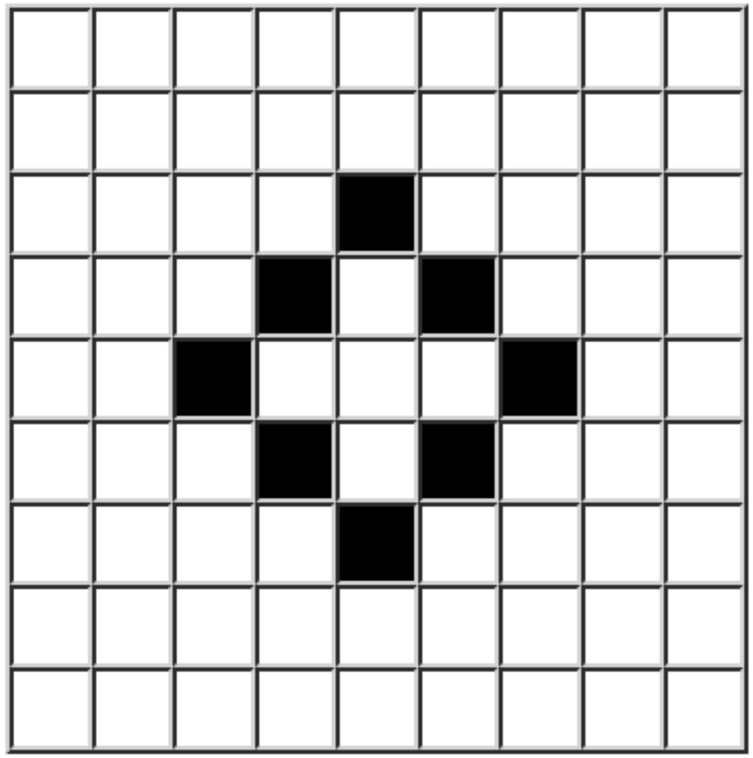
Operation Definitions:

```
u(X,Y) ::
    clear(X) & on(X,Y)
    ==> ~on(X,Y) & table(X) & clear(Y)

s(X,Y) ::
    table(X) & clear(X) & clear(Y)
    ==> ~table(X) & ~clear(Y) & on(X,Y)
```

The Game of Life

World



Rules of the Game

(1) Any *live* cell with *two or three* live neighbors lives on to the next generation.

(2) Any *live* cell with *fewer than two* live neighbors dies (as if caused by underpopulation).

(3) Any *live* cell with *more than three* live neighbors dies (as if by overpopulation).

(4) Any *dead* cell with *exactly three* live neighbors becomes a live cell (as if by reproduction).

Vocabulary

Symbols: `c11, c12, …`

Unary Predicates:

`on` - cell is live

`cell` - true of cells

Binary Predicates:

`neighbor` - cells are neighbors

Starvation

Any *live* cell with *fewer than two* live neighbors dies.

```
tick ::
 on(Y) & countofall(X,neighbor(X,Y)&on(X),0)
 ==> ~on(Y)

tick ::
 on(Y) & countofall(X,neighbor(X,Y)&on(X),1)
 ==> ~on(Y)
```

Overcrowding

Any *live* cell with *more than three* live neighbors dies.

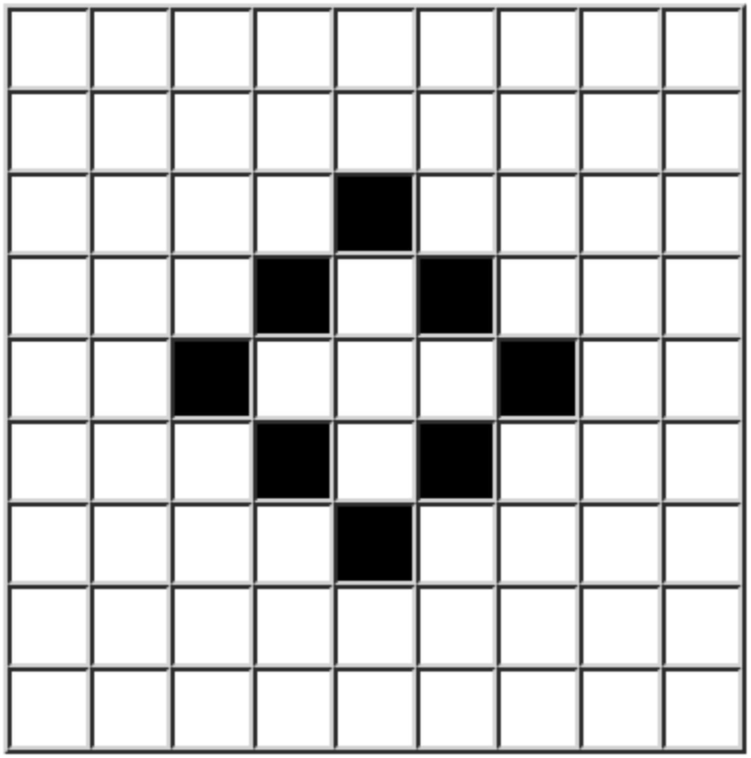
```
tick ::
  on(Y) &
  countofall(X,neighbor(X,Y)&on(X),N) &
  leq(4,N)
  ==> ~on(Y)
```

Transition Rules

Any *dead* cell with *exactly three* live neighbors becomes live.

```
tick ::
  cell(Y) & ~on(Y) &
  countofall(X,neighbor(X,Y)&on(X),3)
  ==> on(Y)
```

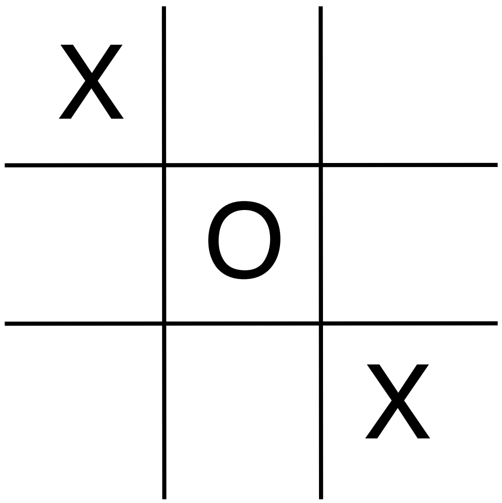
Example



Demonstration

Tic Tac Toe

States



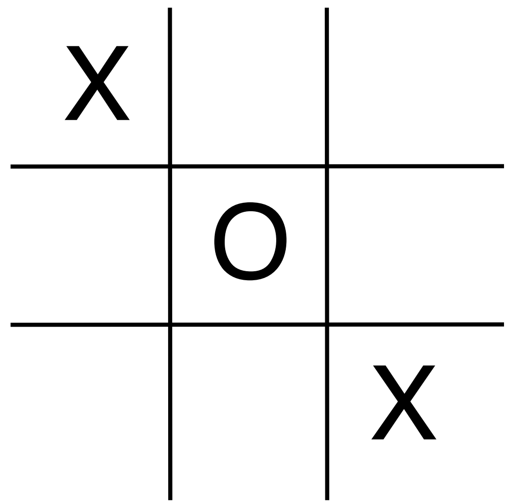

```
cell(1,1,x)
cell(1,2,b)
cell(1,3,b)
cell(2,1,b)
cell(2,2,o)
cell(2,3,b)
cell(3,1,b)
cell(3,2,b)
cell(3,3,x)
control(o)
```

Legal Moves

```
legal(M,N) :- cell(M,N,b)
```

State:

```
cell(1,1,x)
cell(1,2,b)
cell(1,3,b)
cell(2,1,b)
cell(2,2,o)
cell(2,3,b)
cell(3,1,b)
cell(3,2,b)
cell(3,3,x)
control(o)
```



Legal Moves:

```
mark(1,2)
mark(1,3)
mark(2,1)
mark(2,3)
mark(3,1)
mark(3,2)
```

Actions

```
mark(M,N) ::
   control(Z) ==> ~cell(M,N,b) & cell(M,N,Z)
mark(M,N) ::
   control(x) ==> ~control(x) & control(o)
mark(M,N) ::
   control(o) ==> ~control(o) & control(x)
```

```
cell(1,1,x)
cell(1,2,b)
cell(1,3,b)
cell(2,1,b)
cell(2,2,o)
cell(2,3,b)
cell(3,1,b)
cell(3,2,b)
cell(3,3,x)
control(o)
```

mark(1,3) →

```
cell(1,1,x)
cell(1,2,b)
cell(1,3,o)
cell(2,1,b)
cell(2,2,o)
cell(2,3,b)
cell(3,1,b)
cell(3,2,b)
cell(3,3,x)
control(x)
```

Supporting Concepts

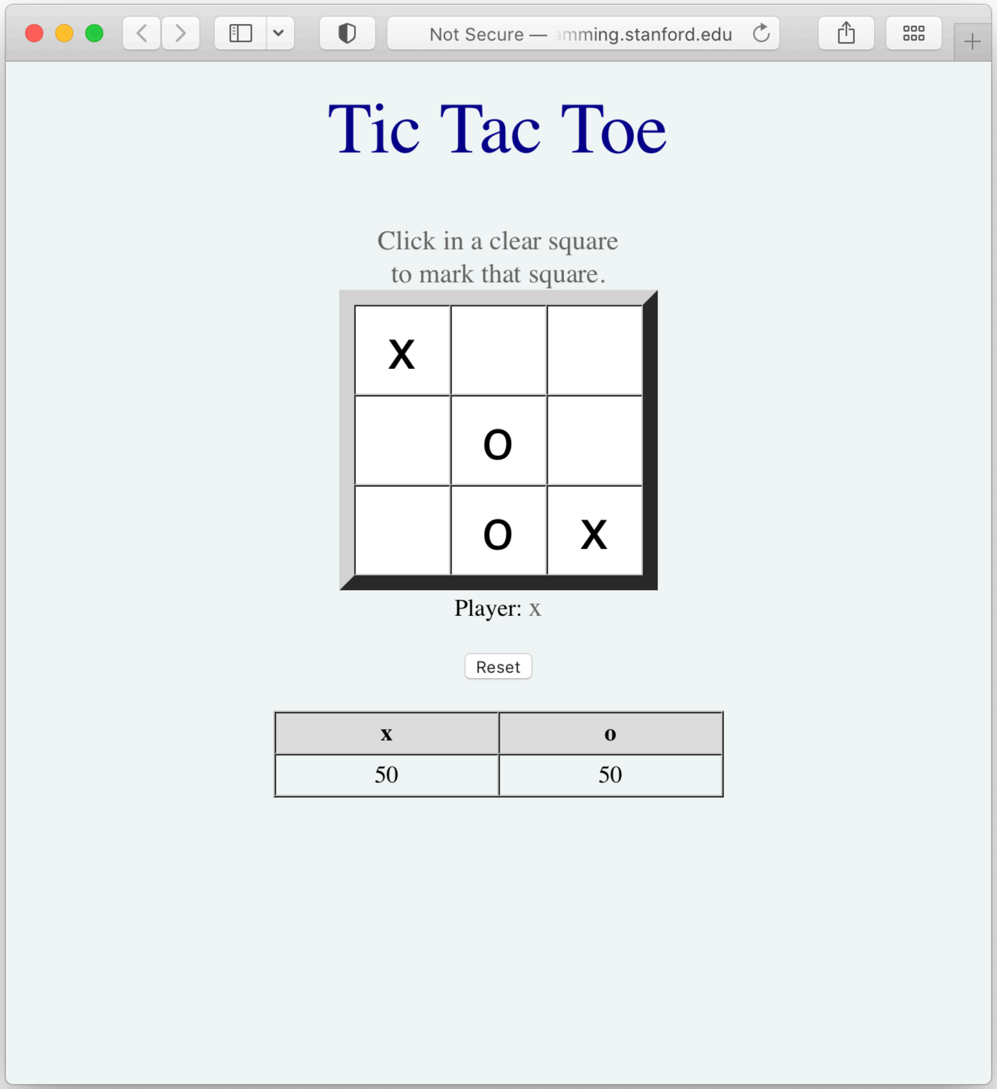
```
row(M,Z) :- cell(M,1,Z) & cell(M,2,Z) & cell(M,3,Z)
col(M,Z) :- cell(1,N,Z) & cell(2,N,Z) & cell(3,N,Z)
diag(Z) :- cell(1,1,Z) & cell(2,2,Z) & cell(3,3,Z)
diag(Z) :- cell(1,3,Z) & cell(2,2,Z) & cell(3,1,Z)

line(Z) :- row(M,Z)
line(Z) :- col(M,Z)
line(Z) :- diag(Z)

win(x) :- line(x)
win(o) :- line(o)

terminal :- win(Z)
terminal :-
  evaluate(countofall([M,N],cell(M,N,b)),0)
```

Example



Demonstration

Assignments

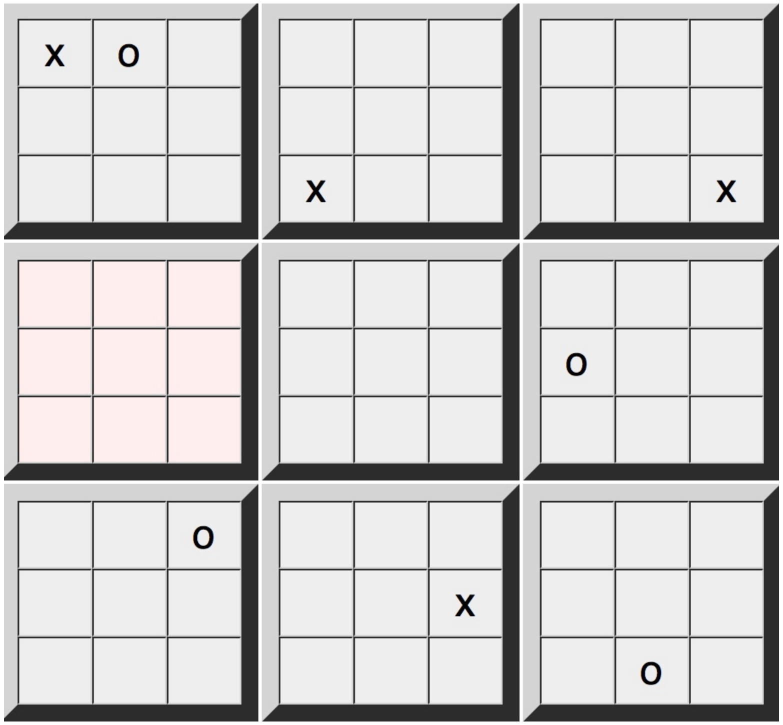
Assignment - Sierra

The goal of this exercise is for you to familiarize yourself with the Sierra capabilities for editing and using action definitions. Go to http://epilog.stanford.edu and click on the Sierra link.

In a separate window, open the documentation for Sierra. To access the documentation, go to http://epilog.stanford.edu, click on Documentation, and then click on the Sierra item on the resulting drop-down menu.

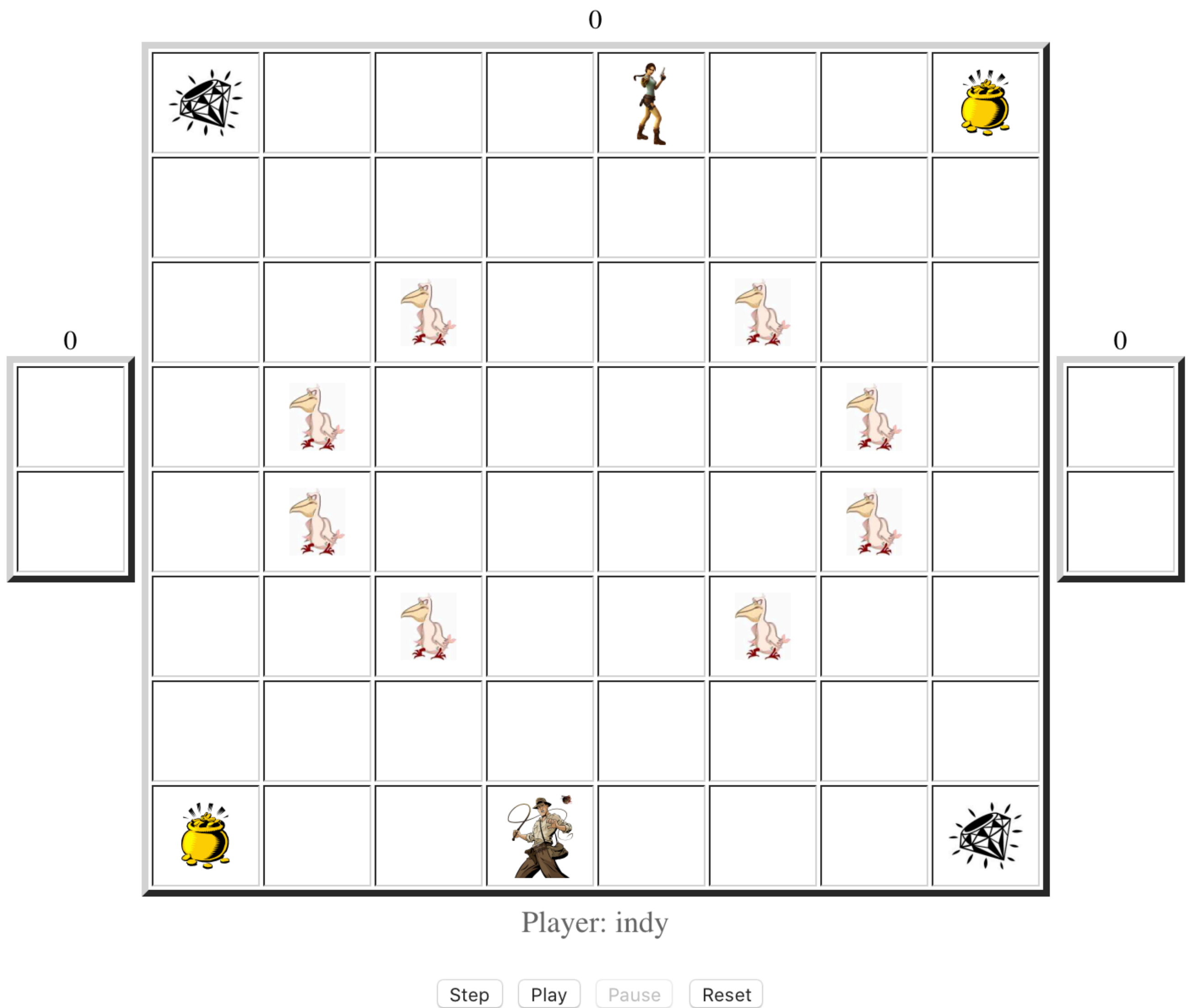
Read though Sections 7 and 8 of the documentation and reproduce the examples in the Sierra window you opened earlier. Once you have done this, experiment on your own. Try different data and different actions.

Assignment - Nineboard Tic Tac Toe

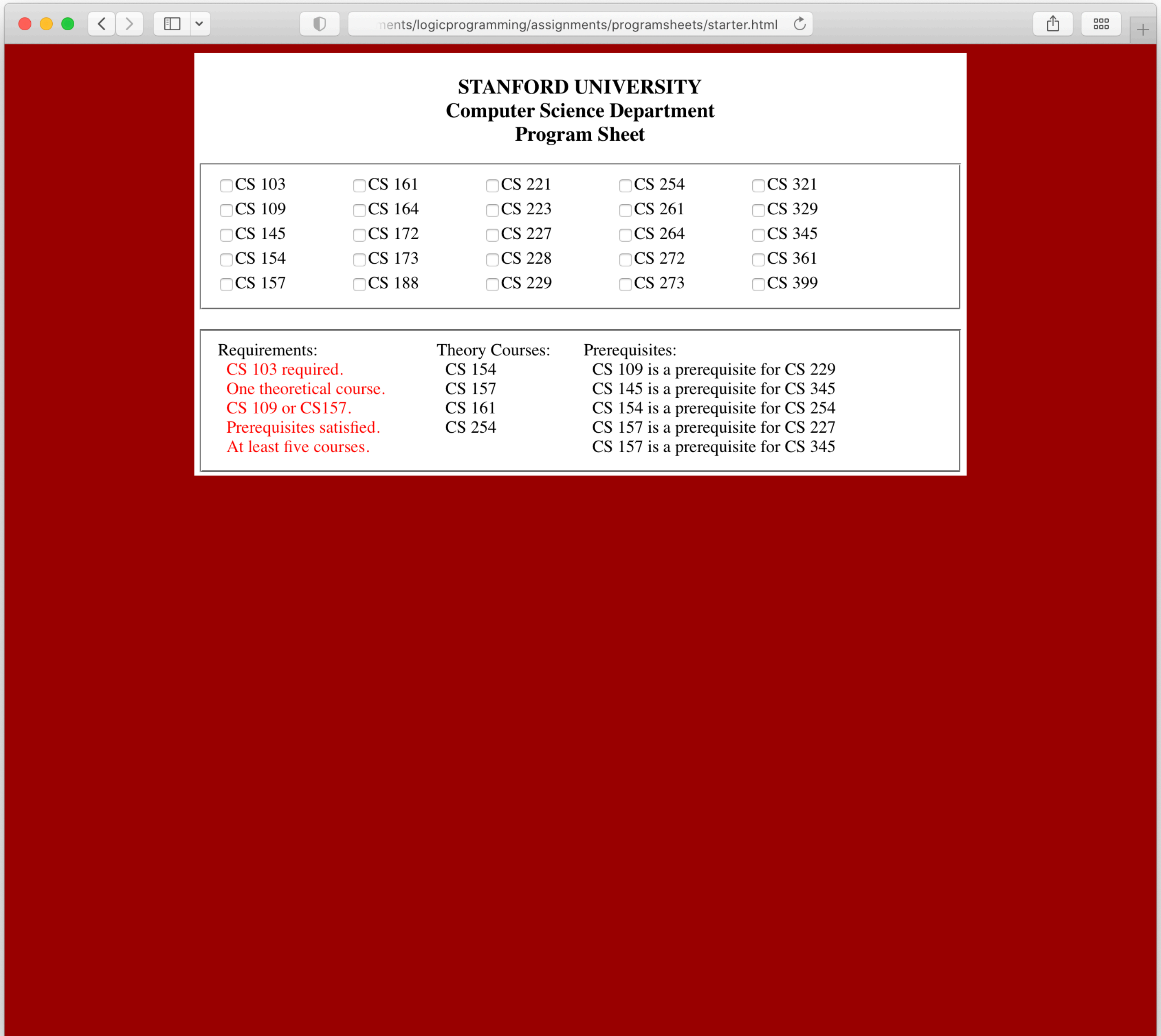


http://logicprogramming.stanford.edu/assignments/nineboard/index.html

Pelican Hunters



http://logicprogramming.stanford.edu/assignments/pelicanhunters/index.html


ents/logicprogramming/assignments/programsheets/starter.html

STANFORD UNIVERSITY
Computer Science Department
Program Sheet

☐CS 103	☐CS 161	☐CS 221	☐CS 254	☐CS 321
☐CS 109	☐CS 164	☐CS 223	☐CS 261	☐CS 329
☐CS 145	☐CS 172	☐CS 227	☐CS 264	☐CS 345
☐CS 154	☐CS 173	☐CS 228	☐CS 272	☐CS 361
☐CS 157	☐CS 188	☐CS 229	☐CS 273	☐CS 399

Requirements:	Theory Courses:	Prerequisites:
CS 103 required.	CS 154	CS 109 is a prerequisite for CS 229
One theoretical course.	CS 157	CS 145 is a prerequisite for CS 345
CS 109 or CS157.	CS 161	CS 154 is a prerequisite for CS 254
Prerequisites satisfied.	CS 254	CS 157 is a prerequisite for CS 227
At least five courses.		CS 157 is a prerequisite for CS 345

http://logicprogramming.stanford.edu/assignments/programsheets/index.html

Schedule

Course	Room	Time
cs151		
cs157		
cs161		

Schedule	g100	g200	g300
morning			
afternoon			
evening			

http://logicprogramming.stanford.edu/assignments/schedule/index.html

Term Project Proposal

[Term Project]()