Inductive Programming
Lecture 8
Game Strategy Induction

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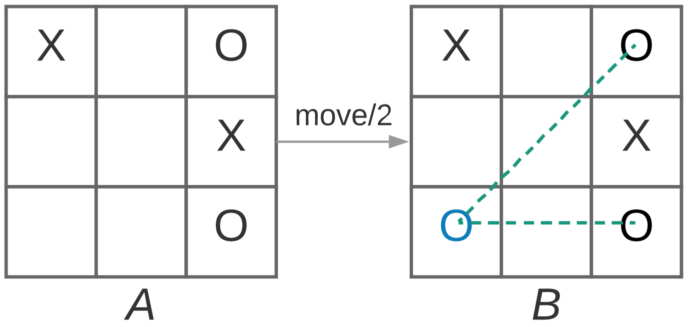
Papers for this lecture

Paper8.1: S.H. Muggleton and C. Hocquette. Machine discovery of comprehensible strategies for simple games using meta-interpretive learning. New Generation Computing, 37:203-217, 2019.

Motivation

- Inductive Programming and AI
- World-class play for Go, Chess, Checkers - AlphaGo (2016) and AlphaZero (2018)
- Deep Reinforcement Learning
- Poor Data Efficiency and Human Comprehensibility
- Meta-Interpretive Game Ordinator (MIGO)
- Minimax Evaluable games - Noughts-and-Crosses and Hexapawn

Noughts and Crosses




```
win_2(A,B):-win_2_1_1(A,B),not(win_2_1_1(B,C)).
win_2_1_1(A,B):-move(A,B),not(win_1(B,C)).
win_1(A,B):- move(A,B),won(B).
```

Related work

Reinforcement Learning World's first reinforecment learning, MENACE (Michie, 1963) learned noughts-and-crosses using matchboxes, punishment and reward beads. HER (Gardner, 1962) for Hexapawn.

Chess endgame strategies Learn minimax depth-of-win using ID3 (Shapiro,Niblett, 1982; Quinlan, 1983) and ILP (Bain Muggleton, 1995).

Q-learning Learn optimal policy (Watkins, 1989). Asymptotic convergence proved (Watkins,Dayan, 1992).

Relational Reinforcement Learning States and actions represented relationally (Dzeroski et al, 2001). Single agent learning problems.

Deep Q-learning Extension of Q-learning with deep convolutional neural network (Mnih et al, 2015). Atari 2600 games. Also AlphaGo (Silver et al, 2016) and AlphaZero (Silver et al, 2018).

Credit assignment problem

Learning by playing Learner evaluates success from outcomes of games.

Credit assignment What is reward for individual moves?

Reinforcement Learning Assign reward to individual moves based on a delay function. Rewards used to update parameters across all board states in game. The number of board states for Noughts-and-Crosses is 10^5; Chess is 10^{45}; Go is 10^{100}.

Exploration vs exploitation Step size $\in [0, 1]$ is degree new information overides old.

Discount factors $\gamma \in [0, 1]$ is importance of future rewards.

Function approximation Deal with larger problem by approximating function over a continuous state space. eg using Convolution Neural Network.

Credit assignment - MIGO

Outcome $Outcome(P, G) \in \{won, drawn, lost\}$ where $won \succ drawn \succ lost$

Play Learner P_1 plays against opponent P_2 which follows minimax strategy.

Selection Game starts from a randomly chosen initial board B.

Lemma 1 The outcome of P_1 monotonically decreases during a game.

Theorem 2 If the outcome is won for P_1, then every move of P_1 is a positive example for the task of winning.

Theorem 3 If S_W accurate strategy and $Outcome(S_W, G) \neq won$ and $Outcome(P_1, G) = drawn$ then every move of P_1 is a positive example for the task of drawing.

MIGO algorithm - Dependency Learning

Input: Positive examples for win_k and draw_k

Output: Strategy for win_k and draw_k

```
for k in [1,Depth] do
    for each example of win_k/2 do
        one shot learn a rule and add it to the BK
    end for
    Learn win_k/2 and add it to the BK
end for
for k in [1,Depth] do
    for each example of draw_k/2 do
        one shot learn a rule and add it to the BK
    end for
    Learn draw_k/2 and add it to the BK
end for
```

MIL representation

Metarules

Name	**Metarule**
postcond	$P(A,B) \leftarrow Q(A,B), R(B).$
negation	$P(A,B) \leftarrow Q(A,B), not(R(B,C)).$

Board state

Pair $s(B,P)$ where board B and player P.

Primitives

Predicate	**Call**
Move	move(S_1, S_2)
Won	won(S)
Drawn	drawn(S)

Game evaluation - minimax regret

Defn 3.4 The **minimax regret** of game G is the difference between minimax outcome of the initial position in G and actual outcome of G.

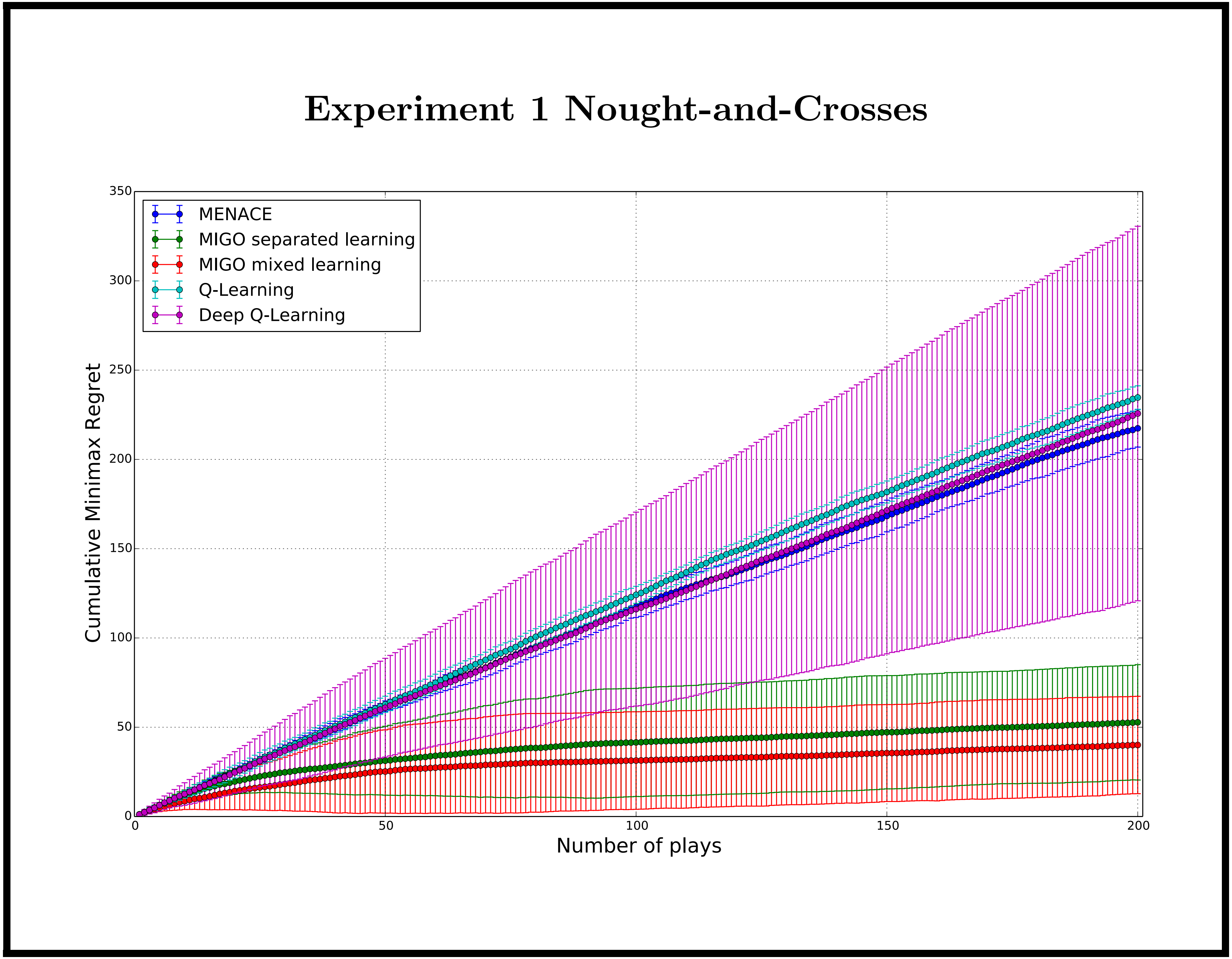
Cumulative minimax regret The sum of minimax regret over a sequence of games. This is an objective measure of performance for competing strategies.

Database Minimax database computed beforehand.

Experiment 1 - Comparison
Cumulative Minimax Regret

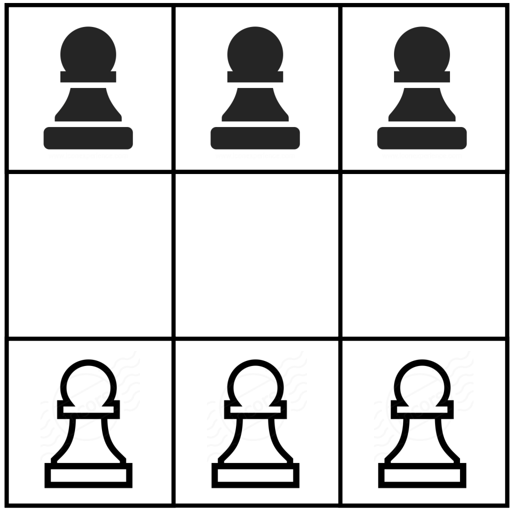
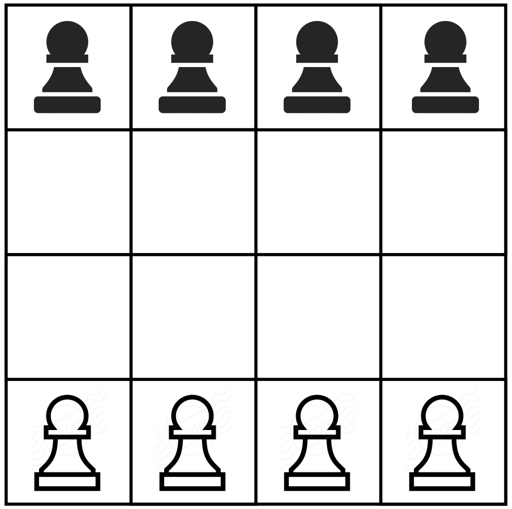
Null Hypothesis 1 MIGO cannot converge faster than MENACE/HER, Q-learning and Deep Q-learning for learning optimal two-player game strategies.

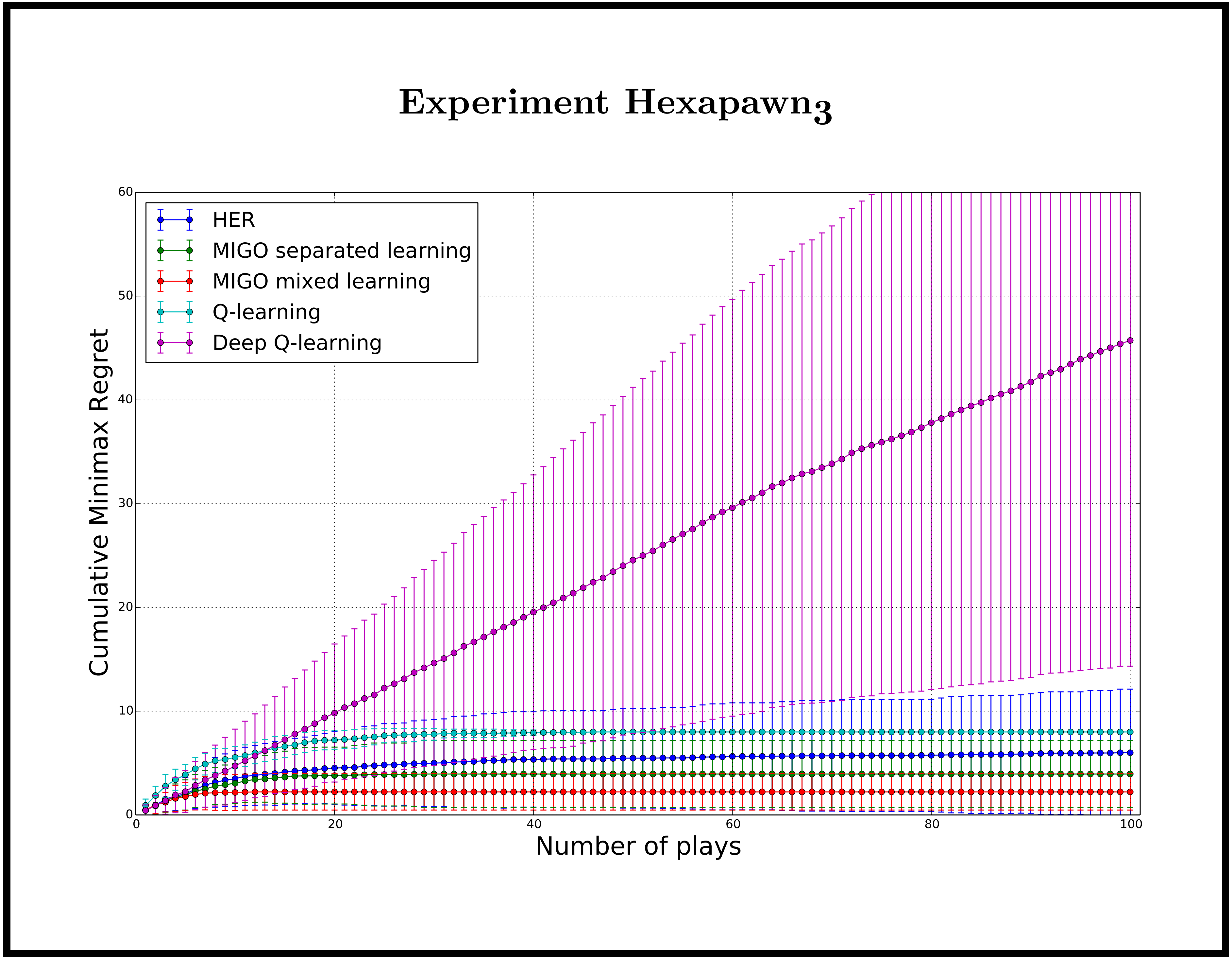
Code for these experiments available at
https://github.com/migo19/migo.git



Experiment 1 Nought-and-Crosses
MENACE
MIGO separated learning
MIGO mixed learning
Q-Learning
Deep Q-Learning
Cumulative Minimax Regret
Number of plays
0
50
100
150
200
250
300
350

Hexapawn

Hexapawn$_3$	Hexapawn$_4$
	


Experiment Hexapawn$_3$
HER
MIGO separated learning
MIGO mixed learning
Q-learning
Deep Q-learning
Cumulative Minimax Regret
Number of plays
0
10
20
30
40
50
60
0
20
40
60
80
100

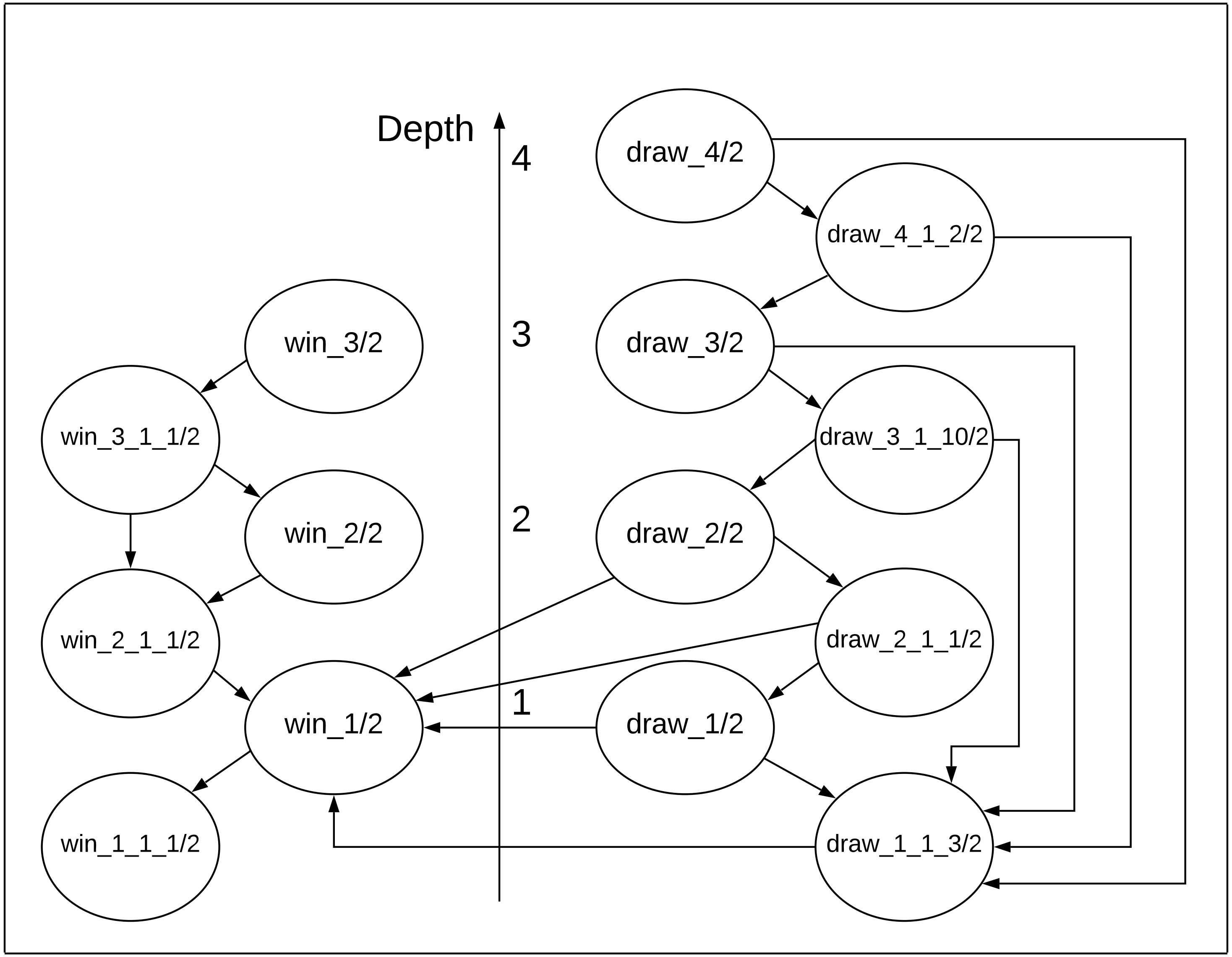
Mean CPU seconds per iteration

	OX	Hexapawn$_3$	Hexapawn$_4$
MIGO mixed learning	$1.5.10^{-1}$	$3.0.10^{-3}$	3.9
MIGO separated learning	$8.9.10^{-2}$	$2.8.10^{-3}$	3.8
MENACE / HER	$1.5.10^{-3}$	$2.7.10^{-4}$	/
Q-Learning	$2.3.10^{-1}$	$1.9\ .10^{-3}$	$2.7\ .10^{-1}$
Deep Q-Learning	$2.4.10^{-1}$	$1.7.10^{-2}$	$2.1\ .10^{-1}$

Learned rules

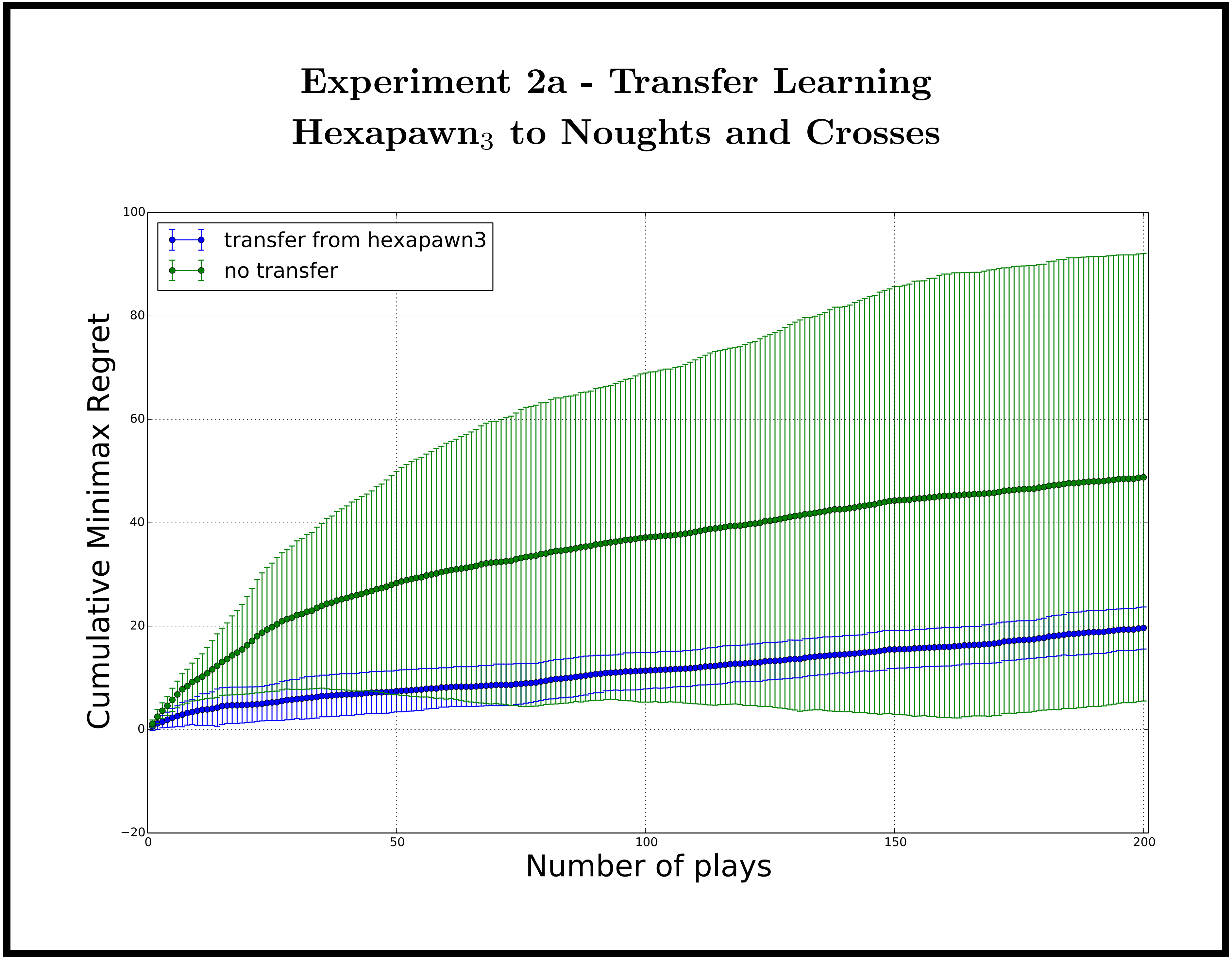
Depth	Rule
1	win_1(A,B):-win_1_1_1(A,B),won(B). win_1_1_1(A,B):-move(A,B),won(B).
	draw_1(A,B):-draw_1_1_3(A,B),not(win_1(B,C)). draw_1_1_3(A,B):-move(A,B),not(win_1(B,C)).
2	win_2(A,B):-win_2_1_1(A,B),not(win_2_1_1(B,C)). win_2_1_1(A,B):-move(A,B),not(win_1(B,C)).
	draw_2(A,B):-draw_2_1_1(A,B),not(win_1(B,C)). draw_2_1_1(A,B):-draw_1(A,B),not(win_1(B,C)).
3	win_3(A,B):-win_3_1_1(A,B),not(win_3_1_1(B,C)). win_3_1_1(A,B):-win_2_1_1(A,B),not(win_2(B,C)).
	draw_3(A,B):-draw_3_1_10(A,B),not(draw_1_1_12(B,C)). draw_3_1_10(A,B):-draw_2(A,B),not(draw_1_1_12(B,C)).
4	draw_4(A,B):-draw_4_1_2(A,B),not(draw_1_1_12(B,C)). draw_4_1_2(A,B):-draw_3(A,B),not(draw_1_1_12(B,C)).

Calling diagram

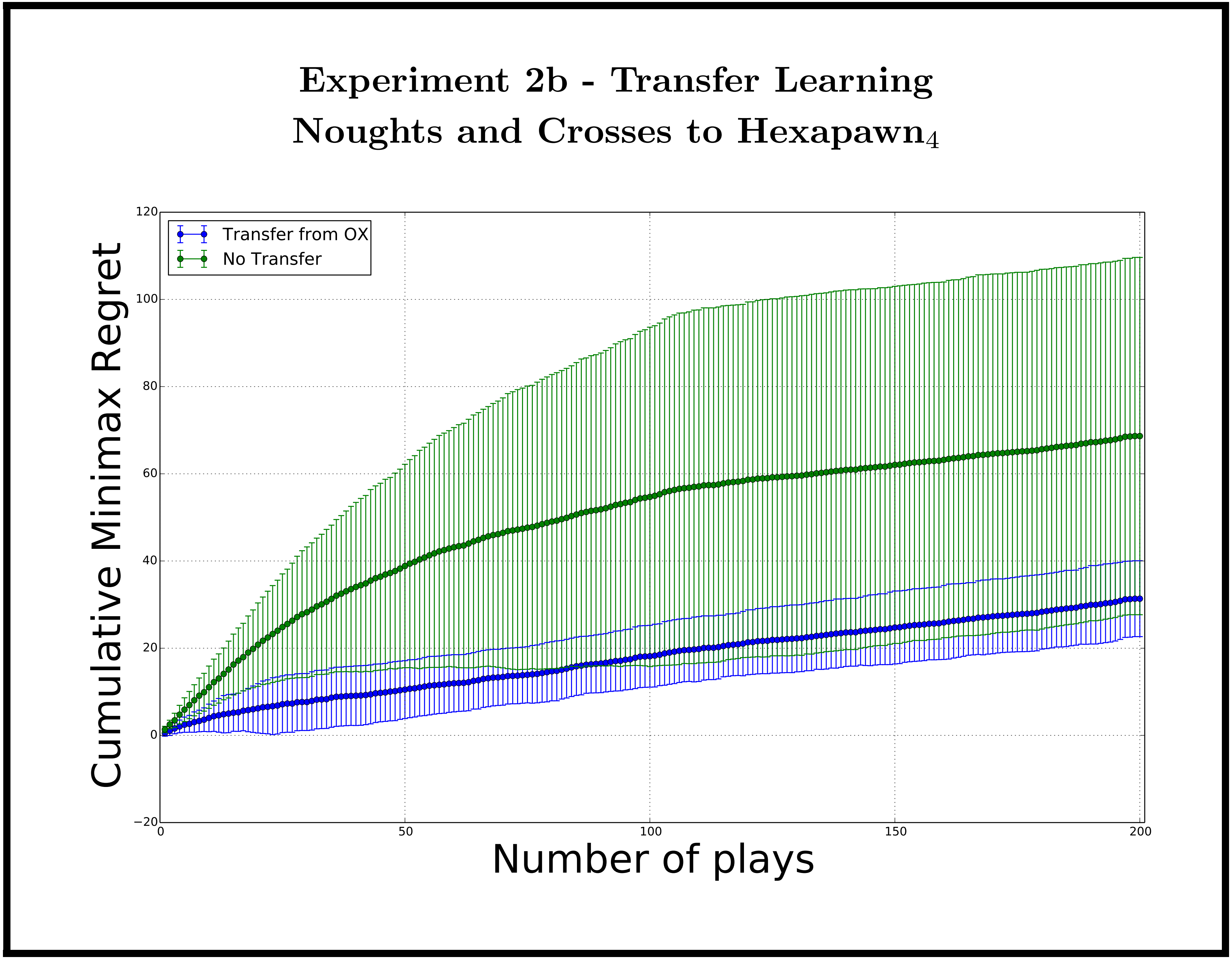


Experiment 2

Null Hypothesis 2 MIGO cannot transfer the knowledge learned during a previous task to a more complex game.



Experiment 2a - Transfer Learning
Hexapawn$_3$ to Noughts and Crosses
transfer from hexapawn3
no transfer
Cumulative Minimax Regret
Number of plays
100
80
60
40
20
0
−20
0
50
100
150
200



Experiment 2b - Transfer Learning
Noughts and Crosses to Hexapawn$_4$
Cumulative Minimax Regret
Number of plays
Transfer from OX
No Transfer
120
100
80
60
40
20
0
−20
0
50
100
150
200

Summary

- MIGO Meta-Interpretive Inductive Programming for two-player-games.
- Novel approach to Credit Assignment Problem.
- Lower Cumulative Minimax Regret than to Deep and classic Q-Learning.
- Strategies transferable to more complex games.
- Over-generalisation since learning from positive example only.
- Running time scales badly with large numbers of board states.
- Optimise running times using Metaopt.
- Assumes optimal opponent - relax assumptions and use self-play.
- Need to assess comprehensibility of strategies. Michie's Ultra-Strong Machine Learning.