Inductive Programming
Lecture 5
Induction of Efficient Programs

Stephen Muggleton
Department of Computing
Imperial College, London and
University of Nanjing

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Change to lecture times

Lecture 6 will be held in person at 1610 on Thursday 9th November.

Lecture 7 and 8 will be held in person at 1610 on Monday 13th and 20th November.

Papers for this lecture

Paper5.1: A. Cropper and S.H. Muggleton. Learning efficient logical robot strategies involving composable objects. In Proceedings of the 24th International Joint Conference Artificial Intelligence (IJCAI 2015), pages 3423-3429. IJCAI, 2015.

Paper5.2: A. Cropper and S.H. Muggleton. Learning efficient logic programs. Machine Learning, 108:1063-1083, 2019.

Motivation

- Inductive Programming
- Few examples per task
- Short programs preferred - Blumer bound
- Are shorter programs always preferable?

Permutation Sort versus Merge Sort

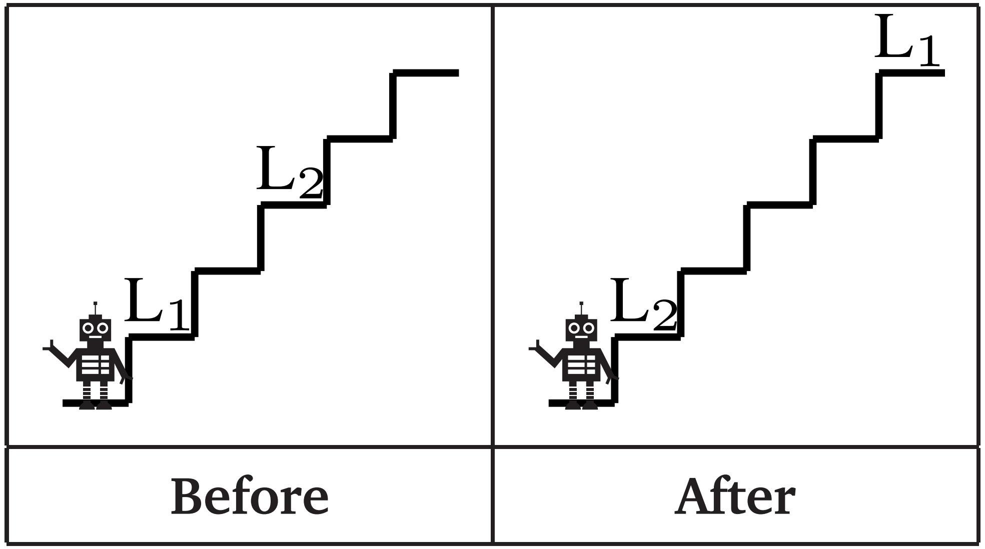
Program size

psort	s(L1,L2) :- permute(L1,L2), sorted(L2).
msort	s([],[]). s([H|T],L) :- sp(H,T,L1,L2), s(L1,L3), s(L2,L4), m(L3,L4,L).

Time complexity

psort	O(n!)
msort	O(nlog(n))

Postman [Paper5.1]



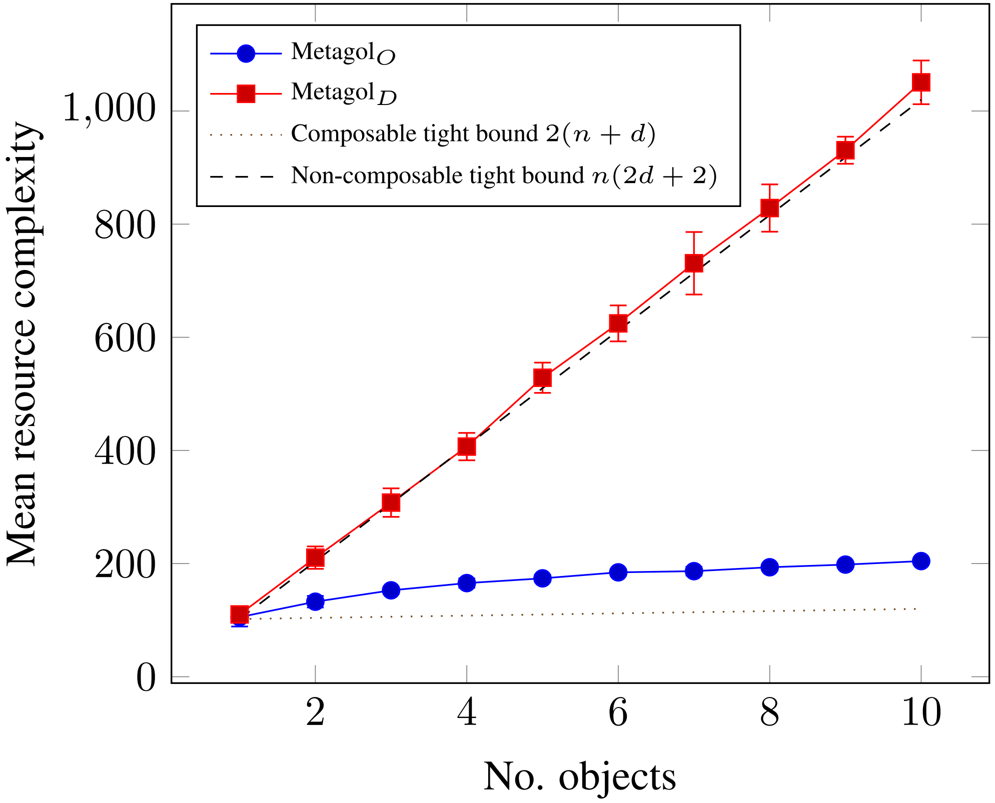
n letters and **d** places for delivery

Postman [Paper5.1]

Metagol$_D$ - $O(nd)$
p(A,B):- p2(A,C), p(C,B).
p(A,B):- p2(A,C), gtb(C,B).
p2(A,B):- p1(A,C), gtb(C,B).
p1(A,B):- fns(A,C), take(C,B).
p1(A,B):- fnr(A,C), give(C,B).

Metagol$_O$ - $O(n+d)$
p(A,B):- p2(A,C), p2(C,B).
p2(A,B):- p1(A,C), p2(C,B).
p2(A,B):- p1(A,C), gtb(C,B).
p1(A,B):- fns(A,C), bag(C,B).
p1(A,B):- fnr(A,C), give(C,B).

Postman mean resource complexity 50 places [Paper5.1]

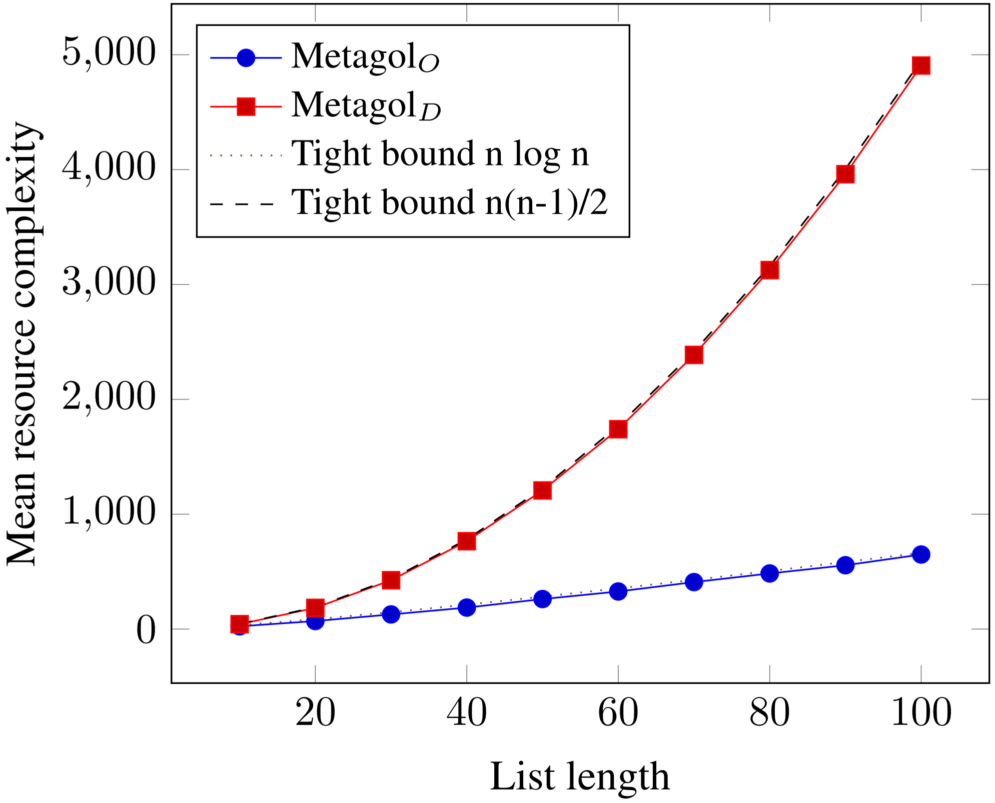


Robot Letter Sorter [Paper5.1]

Metagol$_D$ **-** $O(n^2)$
rs(A,B):- rs1(A,C), rs(C,B).
rs1(A,B):- cmp(A,C), rs1(C,B).
rs1(A,B):- dec(A,C), gst(C,B).
rs(A,B):- rs1(A,C), gst(C,B).

Metagol$_O$ **-** $O(nlog(n))$
rs(A,B):- rs1(A,C), rs(C,B).
rs1(A,B):- pick(A,C), split(C,B).
rs1(A,B):- cmb(A,C), gst(C,B).
rs(A,B):- split(A,C), cmb(C,B).

Robot Letter Sorting mean resource complexity [Paper5.1]



Duplicate Character [Paper5.2]

Examples

```
f([p,r,o,g,r,a,m],r).
f([i,n,d,u,c,t,i,o,n],i).
```

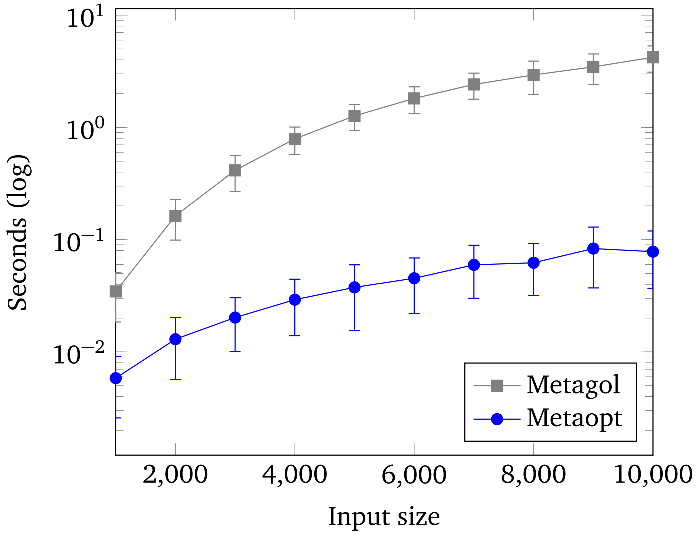
Duplicate Character [Paper5.2]

Metagol$_D$ - $O(n^2)$

```
f(A,B):-head(A,B),f_1(A,B).
f(A,B):-tail(A,C),f(C,B).
f_1(A,B):-tail(A,C),element(C,B).
```

Metaopt - $O(nlog(n))$

```
f(A,B):-msort(A,C),f_1(C,B).
f_1(A,B):-head(A,B),f_2(A,B).
f_1(A,B):-tail(A,C),f_1(C,B).
f_2(A,B):-tail(A,C),head(C,B).
```



Framework - Cost function Φ [Paper5.2]

Metagol_O	$\sum_{e \in E} r(H, e)$
Metaopt	$\sum_{e \in E} \text{treecost}(H, e)$
General ordering	$\prec_{\Phi}$

Framework - Cost minimisation over Version Space [Paper5.2]

Dfn6	Version space $\mathcal{V}_{B,E}$	Hypothesis space consistent with B, E
Dfn7	Cost minimisation	$H \in \mathcal{V}_{B,E}$ and $\forall H' \in \mathcal{V}_{B,E} H \preceq_{\Phi} H'$

Metagol$_O$ and Metaopt algorithm

Cost Minimisation

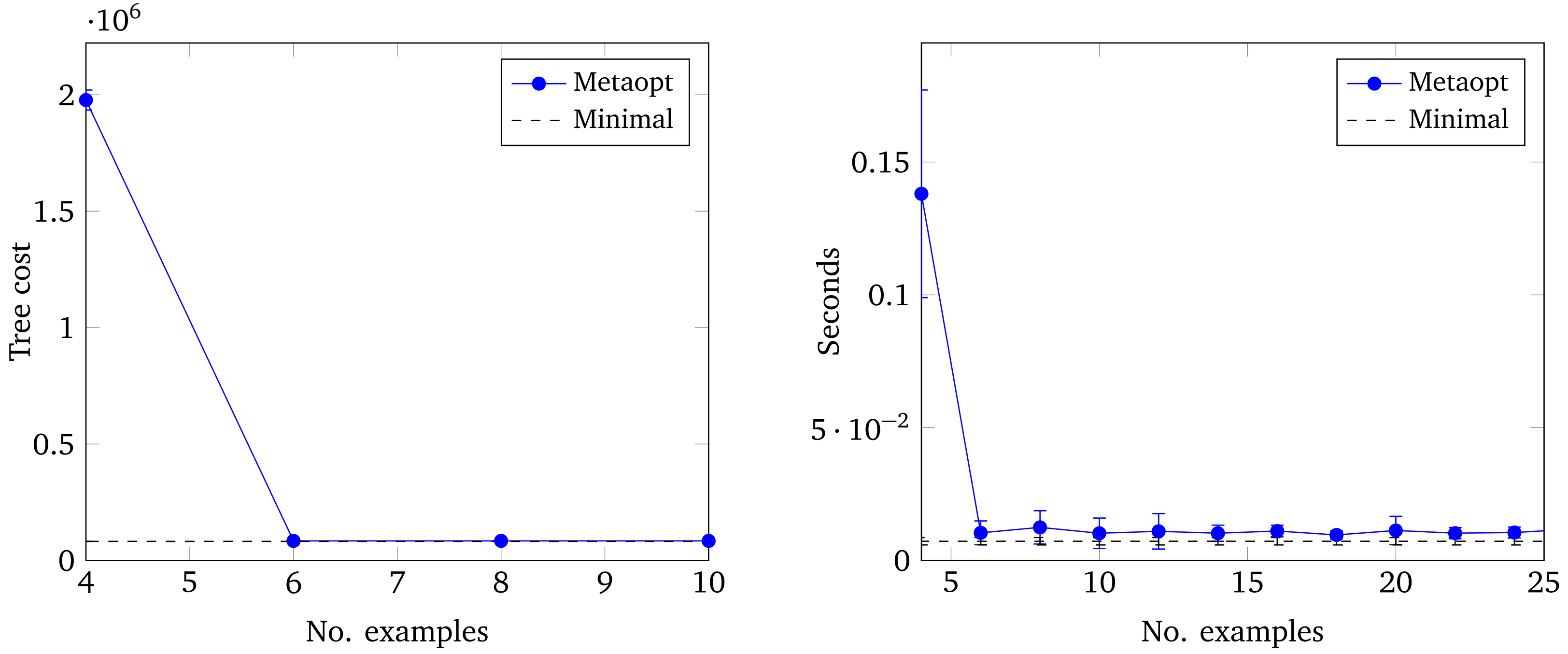
Iteration	Hypothesis
1	$\|\mathrm{H}_1\|$ minimal in $\mathcal{V}_{B,E}$
$i > 1$	$\|H_i\|$ minimal and $H_i \prec_\Phi H_{i-1}$
$i = \mathrm{final}$	$\nexists H_i \; H_i \prec_\Phi H_{i-1}$
Return	$H_{\mathrm{final}-1}$

Convergence theorem [Thm 1 Paper5.2]

Given sufficiently large $|E|$

Metaopt returns $\inf_{\preceq_\Phi} \mathcal{V}_{B,E}$

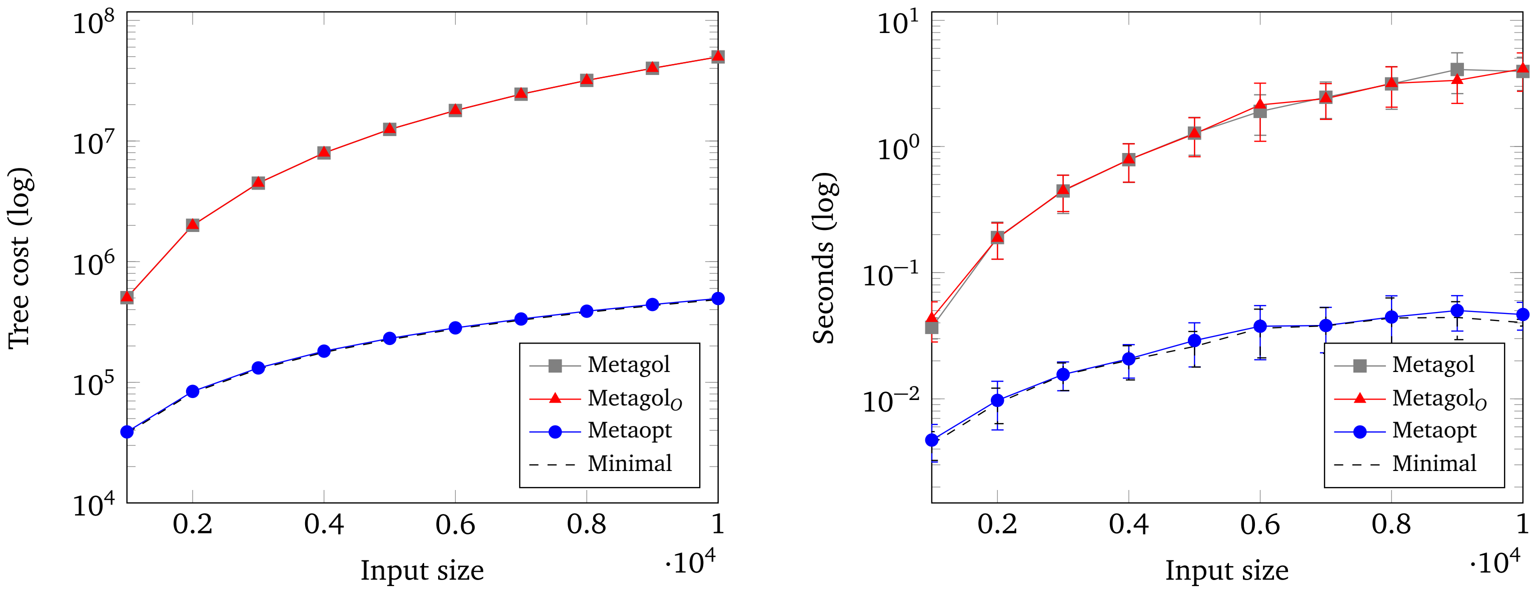
Duplicate Character - Median Tree Costs [Paper5.2]



(a) Tree costs

(b) Program runtimes

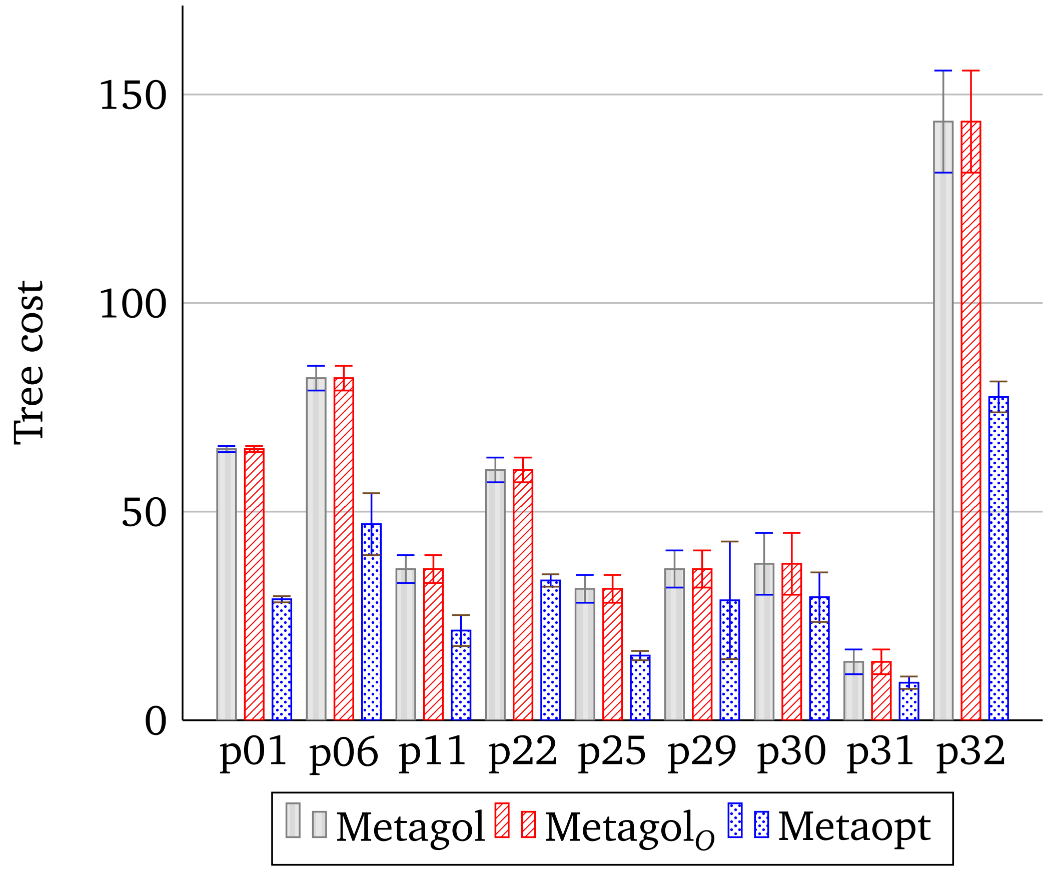
Duplicate Letter - Input size vs Tree Cost [Paper5.2]



(a) Tree costs

(b) Program runtimes

String Transformations - Median Tree Costs [Paper5.2]



Summary

- Shorter programs fewer examples - Blumer bound
- Shorter programs not always most efficient
- Metagol$_O$ minimises robot energy cost
- Metaopt minimises SLD resolutions
- Both find minimal size program first
- Iteratively relax size minimising cost Φ
- Convergence theorem
- Efficiency - Postman, Sorting, Duplicate, String Transform